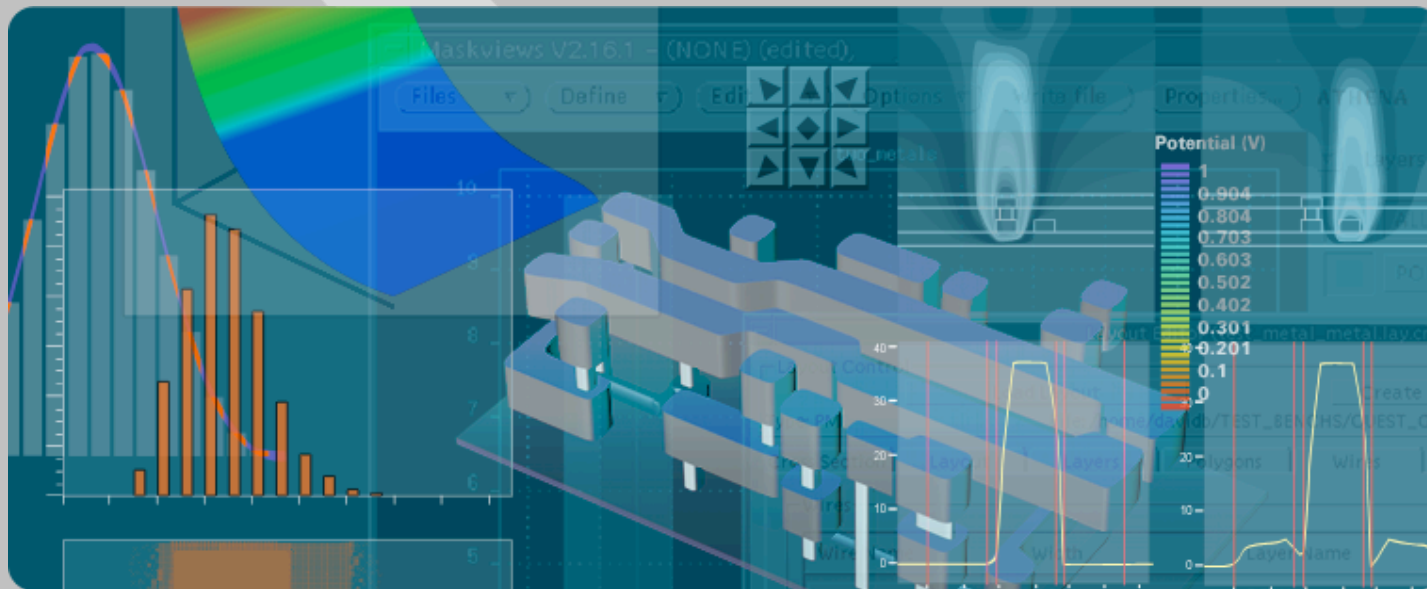


## QUEST/STELLAR Numerical Methods



Advanced Numerical Methods for Parasitic Extraction



## Outline

- Numerical Key Points
- STELLAR
  - Fictitious Domain Method
  - Coupling surface and grid
  - Domain Decomposition and Halo Method
- QUEST
  - Coupling surface and grid
  - Capacitance problem
  - Impedance problem
  - Comparisons with measurements

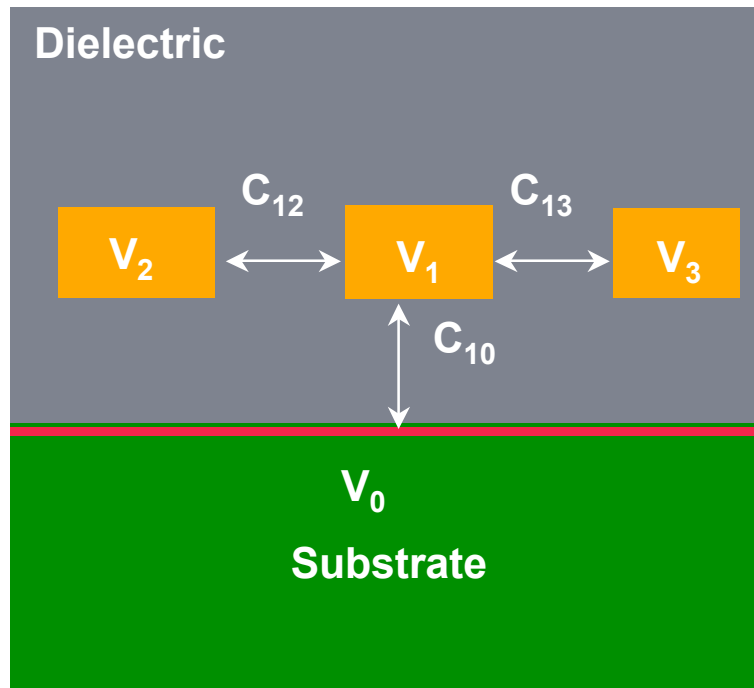


## Numerical Key Points

- 3D Field solver based on Quasi-Static Maxwell Equations
- Use 3D regular grids and surface triangle meshes
  - No 3D Tetrahedron meshing
  - Low memory needs
  - Fast



## STELLAR 3D Capacitance extractor

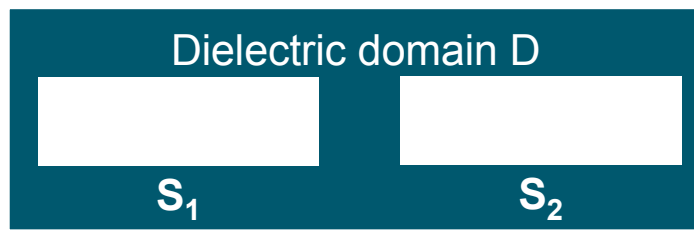


$$Q_i = \sum_{j=0}^N C_{ij} (V_i - V_j)$$



## Domain Problem

Initial Problem

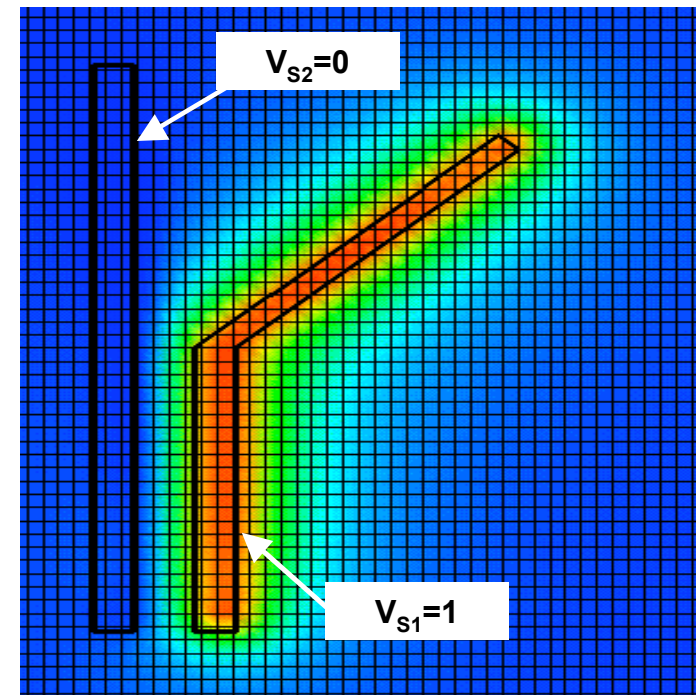
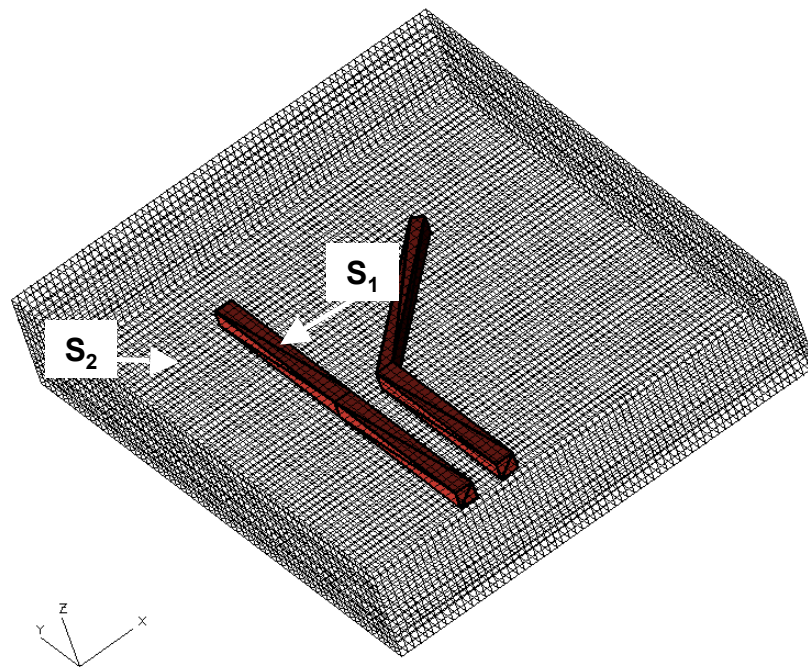
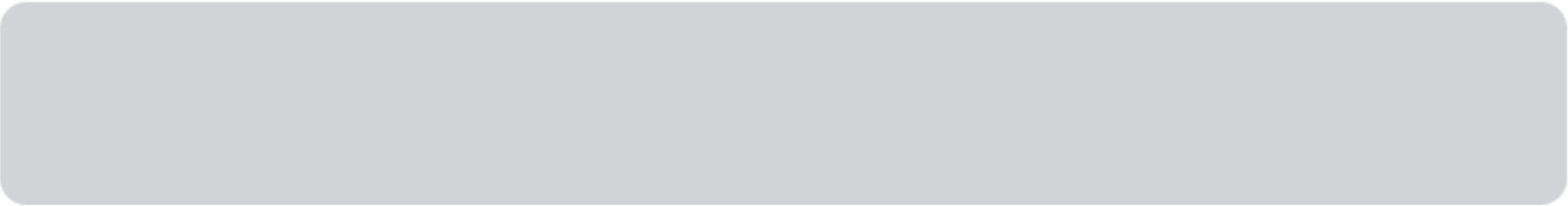


$$\rightarrow \begin{cases} \Delta V = 0 & , \text{On } D \\ V_{S_i} = V_i & , \text{On } S_i \end{cases}$$

Fictitious Domain Problem



$$\rightarrow \begin{cases} \Delta V = q_{S_1} + q_{S_2} & , \text{On } \Omega \\ V_{S_i} = V_i & , \text{On } S_i \end{cases}$$



- $V$ : 3D regular grid
- $q$ : triangle surface mesh



## Fictitious Domain Matrix System

$$\begin{cases} \Delta V - q_{S_1} + q_{S_2} = 0 & , \text{On } \Omega \\ V_{S_i} = V_i & , \text{On } S_i \end{cases}$$

$$\rightarrow \begin{cases} AV_{3D} - BQ_{2D} = 0 \\ B^T V_{3D} + CQ_{2D} = G \end{cases} \rightarrow (B^T A^{-1} B + C) Q_{2D} = G$$

$A^{-1}$  is calculated using FFT Fast Solver

### Size Estimation

$N_{3D}$ : 3D grid nodes

$N_{2D}$ : 2D surface triangle

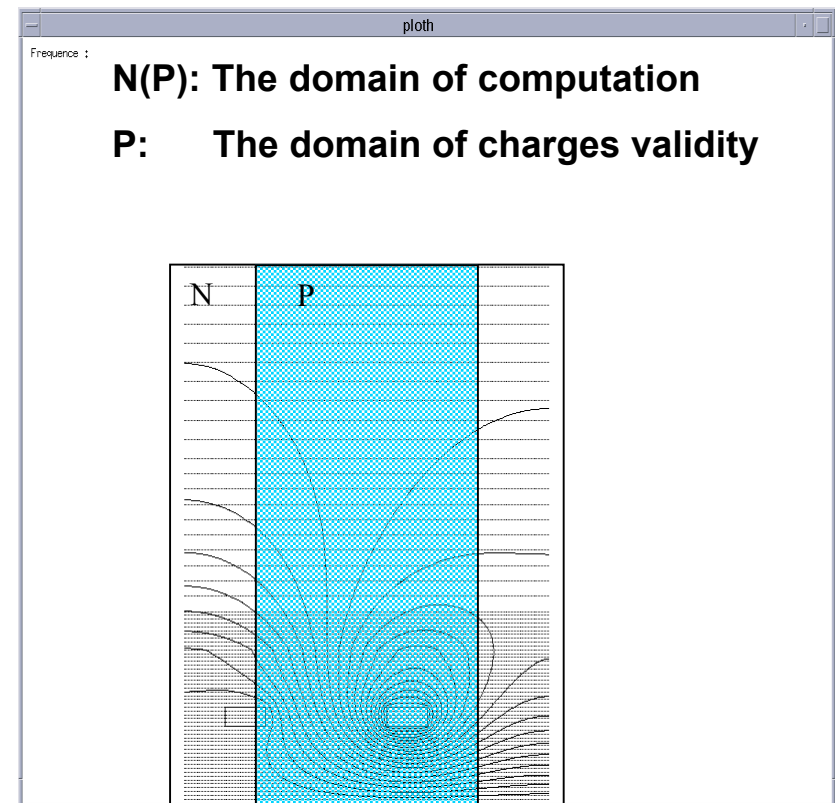
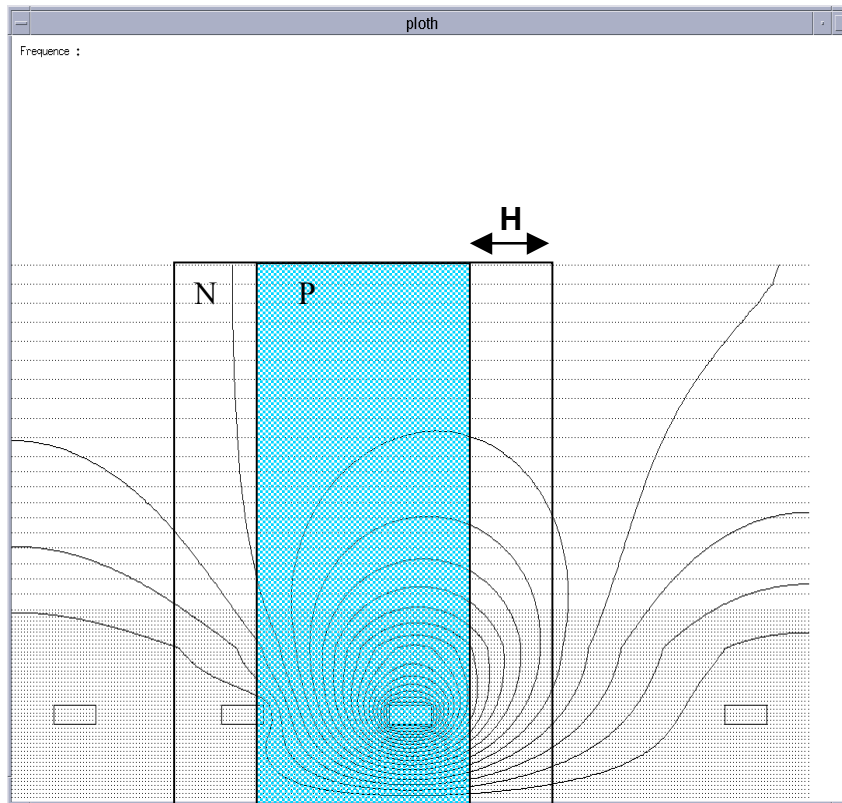
Finite Elements  $30 N_{3D}$

Boundary Elements  $N_{2D}^2$

Fictitious Domain  $2N_{3D} + 10N_{2D}$

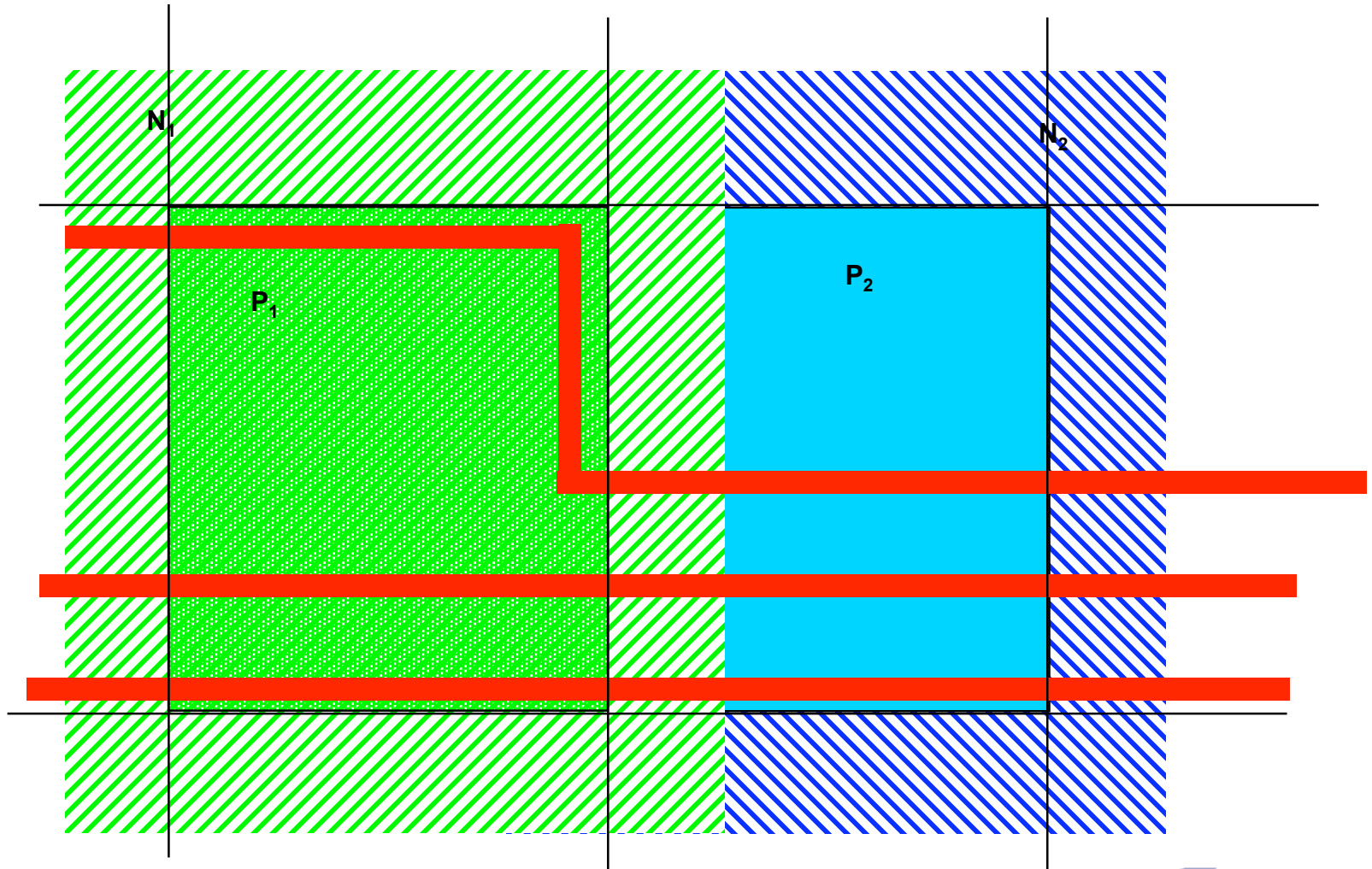


# Domain Decomposition Schema



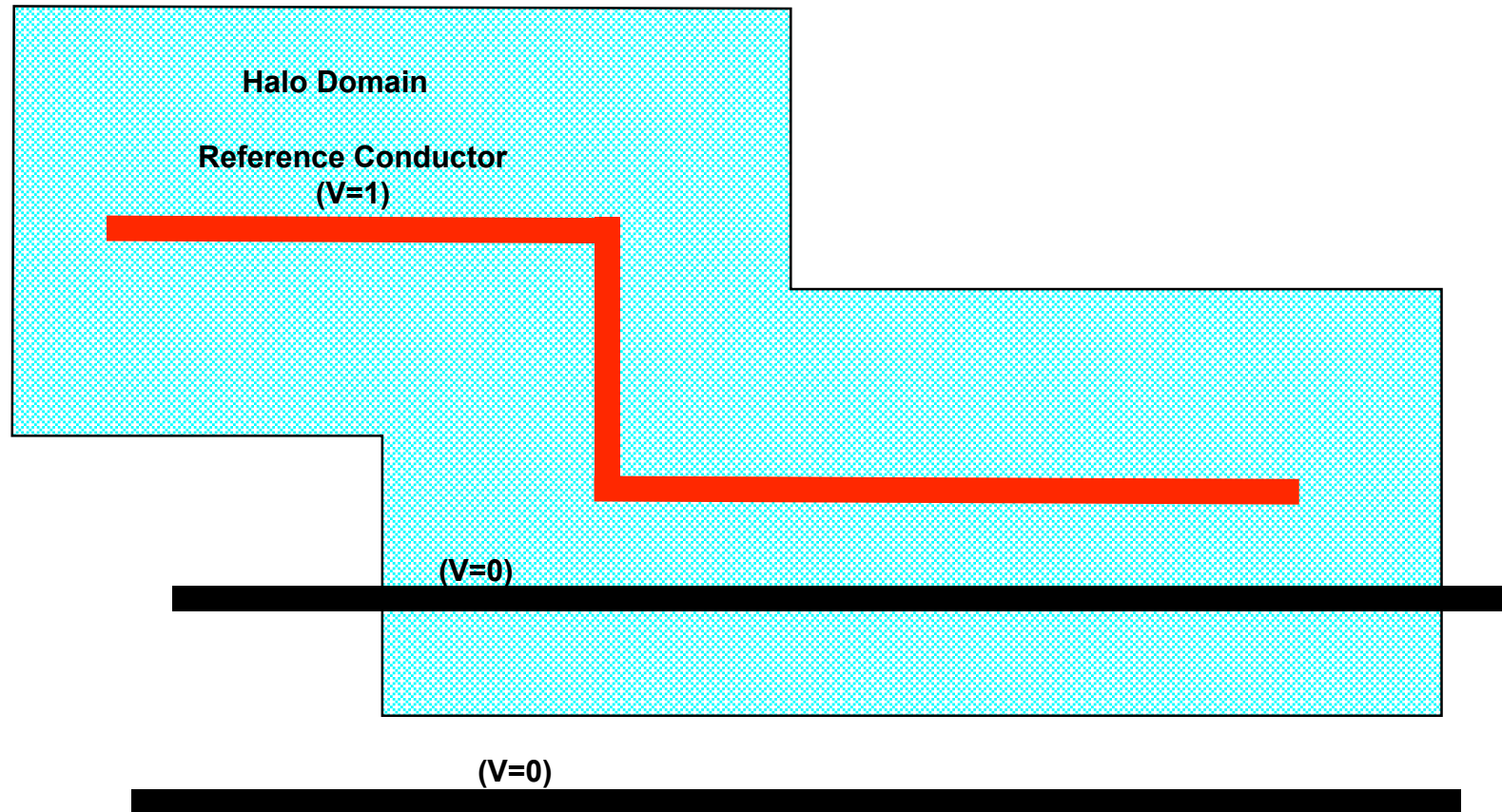


# Domain Decomposition Schema





# Halo Method Schema





## Example of Domain Decomposition

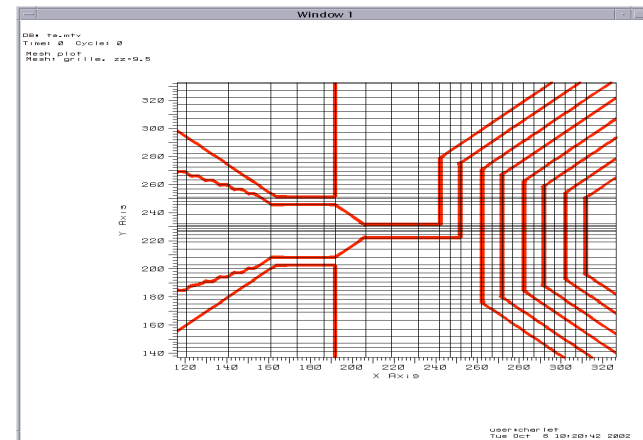
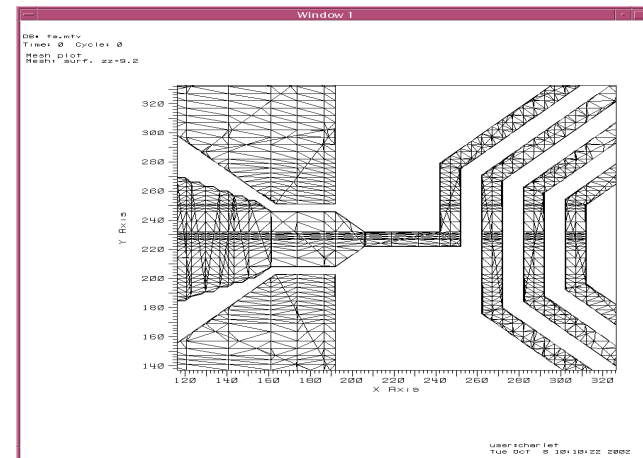


SUN-SPARC-V880 900 MHz	No Decomposition	Decomposition H=8mm
CPU(sec)	1300	600
Mem. (GB)	1.200	0.200
C(F) in/out	14.04 $10^{-14}$	13.80 $10^{-14}$
in/substrate	12.46 $10^{-14}$	12.49 $10^{-14}$
out/substrate	9.63 $10^{-14}$	9.69 $10^{-14}$



# QUEST 3D frequency domain extractor

- Quasi-Static Maxwell Equation
- Surface Triangle Meshe
  - Electric Scalar Potential , Electric Charge
  - Electric Vector Potential
- 3D grid
  - Electric Scalar Potential
  - Magnetic Vector Potential





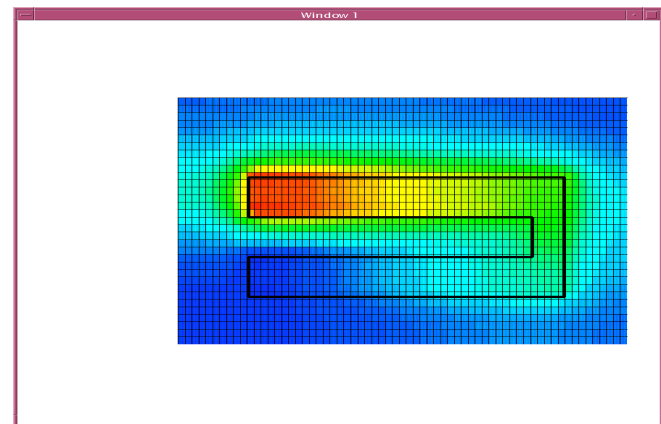
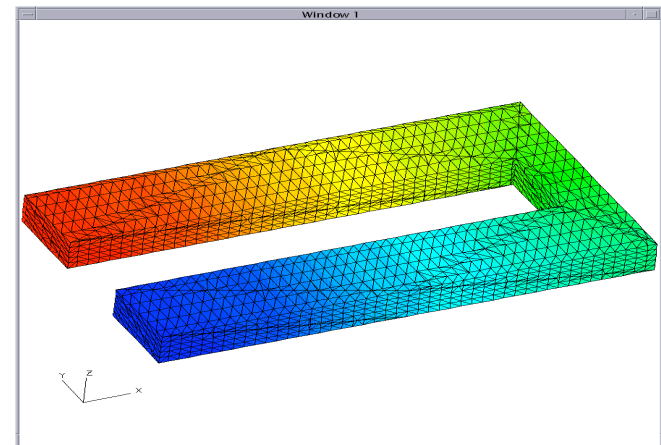
# Capacitance Problem

- Surface Triangle Mesh
  - Local Surface Impedance  $z_s$
  - Electric Scalar Potential  $u_s$
  - Electric Charge  $q_s$

$$\begin{cases} \nabla \cdot z_s^{-1} \nabla u_s = 0 \\ z_s^{-1} \nabla u_s \cdot n_s = j\omega(q_s) \end{cases}$$

- 3D grid
  - Electric Scalar Potential  $u_V$

$$\begin{cases} \nabla \cdot \epsilon \nabla u_V - q_s = 0 \\ u_V = u_s \text{ on } S \end{cases}$$

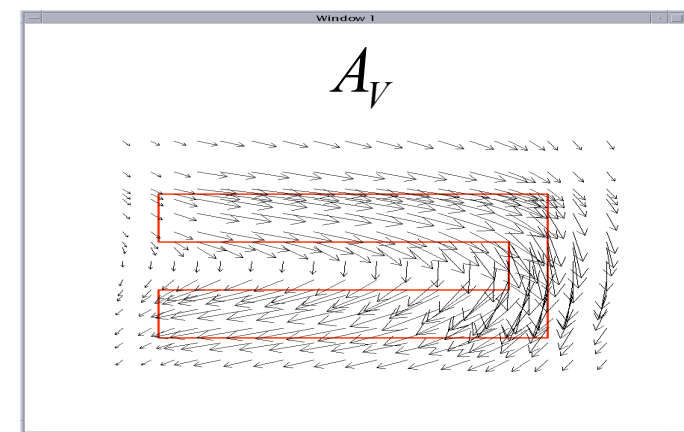
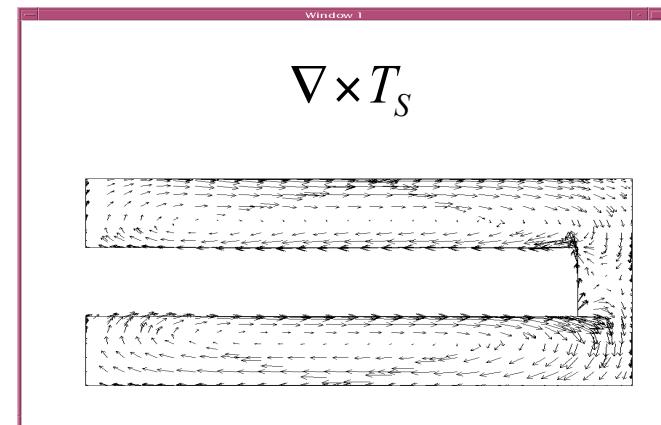




# Impedance Problem

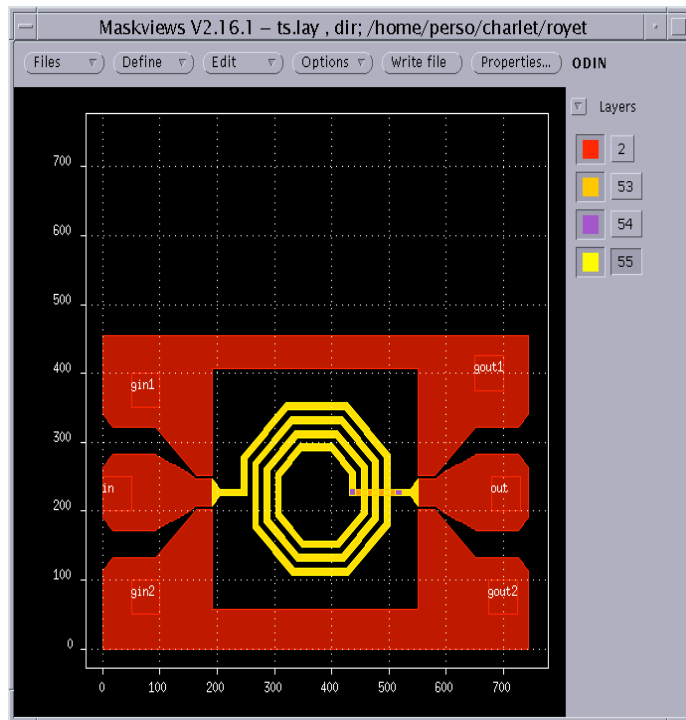
- Quasi-Static Maxwell Equation
- Surface Triangle Mesh
  - Surface conductivity  $\sigma_s$
  - Electric Scalar Potential  $u_s$
  - Electric Vector Potential  $T_s$
- 3D grid
  - Magnetic Vector Potential  $A_V$

$$\begin{cases} \nabla \times \mu^{-1} \nabla \times A_V + j\omega \nabla \times T_s & = j\omega \sigma_s \nabla u_s \\ j\omega \nabla \times A_V + \nabla \times \cdot \sigma_s^{-1} \nabla \times T_s & = 0 \end{cases}$$

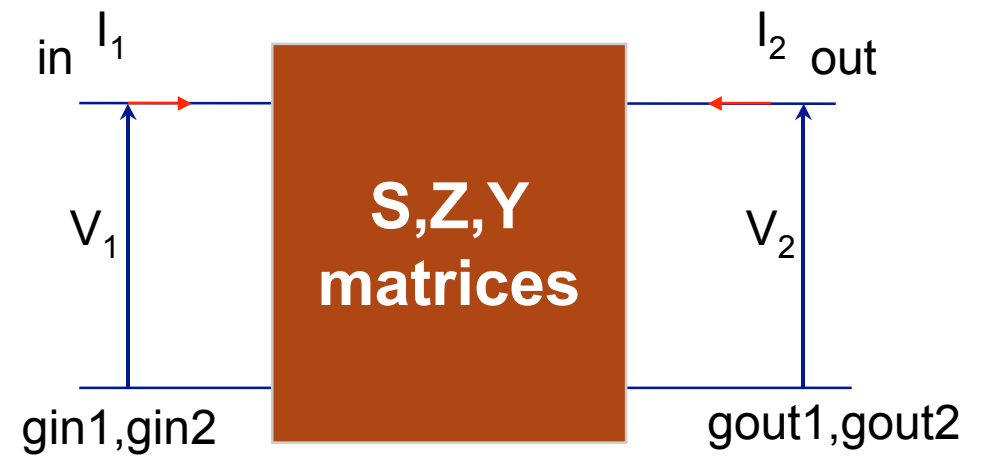




# Comparisons with Measurements (CEA-LETI)

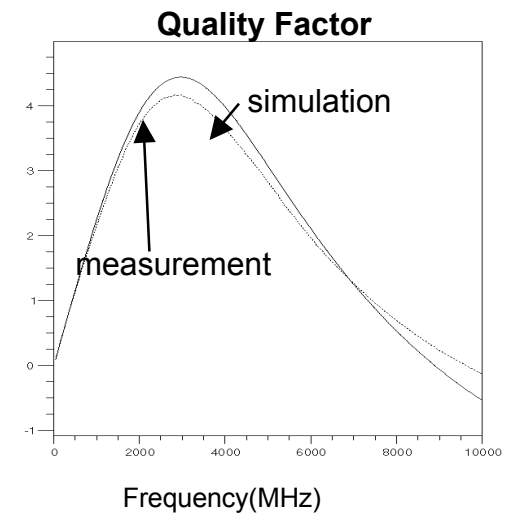
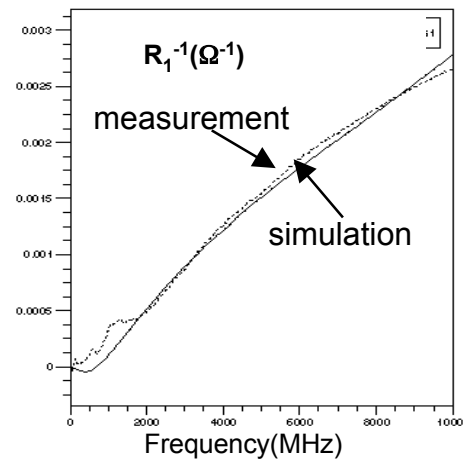
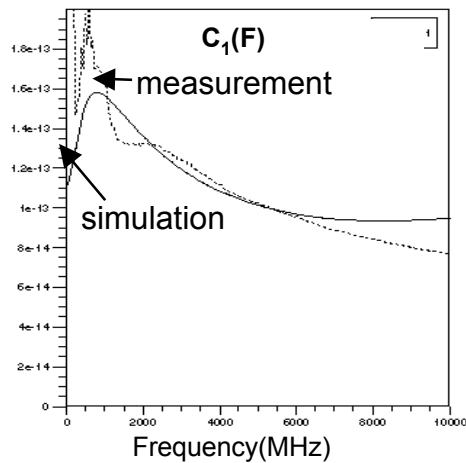
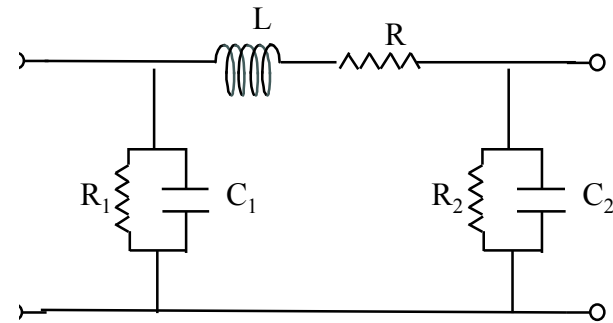
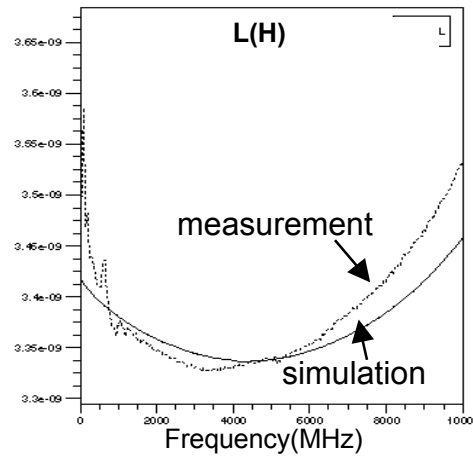
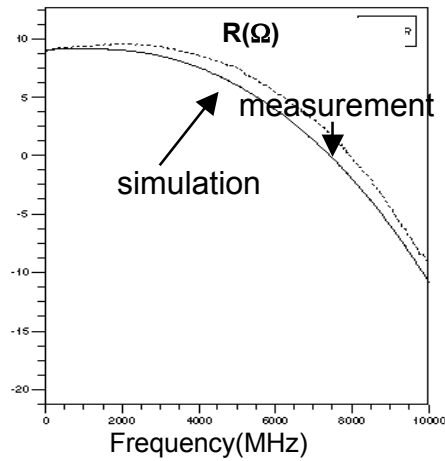


## 2 Ports Model





# Comparisons with Measurements (CEA-LETI)





## Conclusion

- **STELLAR/QUEST Original Numerical Methods**
  - Physics 3D equations
  - Accuracy
  - Low Memory needs
  - Fast
  - Large Geometries