Noise Modeling in MOSFET and Bipolar Devices

Analog/Mixed-Signal Simulation
Noise Modeling

Analog/Mixed-Signal Simulation

MOSFET Noise
Overview

1. Noise Concept

2. MOSFET Noise
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   - Thermal Noise in MOSFET (SPICE 2 & BSIM3)
   - How to modeling for 1/f noise
   - Advanced Noise Model

3. BJT (Bipolar) Noise
   - How to measure 1/f noise for MOSFET and BJT
   - Various Noise in BJT (1/f, Thermal, Shot noise)
   - Noise model equation
   - How to modeling for 1/f noise
Noise Concept

1. Flicker Noise (1/f noise, pink noise)
   Random trapping and detrapping of the mobile carriers in the channel and within the gate oxide (McWhorther’s model, Hooges’ model)

2. Shot Noise
   Every reverse biased junction generates shot noise which is caused by random carriers across the junction.

3. Thermal Noise (Johnson noise, Nyquist noise)
   Random thermally excited vibration of the charge carriers.

4. Generation/Recombination Noise
   Trapping centers in the bulk of the device can cause generation/recombination noise.
MOS Flicker Noise or 1/f Noise

- **McWhorther’s model:** noise is caused by the random Trapping and detrapping of the mobile carriers in the channel
- **Hooges’ model:** the flicker noise is attributed to mobility fluctuation

MOS equivalent circuit for Noise model
McWhorther’s model (1/f noise)

- Carrier number fluctuation theory known also as the trapping-detrapping model, proposed by McWhorther. But these fluctuations can also induce fluctuation in the channel mobility of the remaining carriers in the channel since the traps act as coulombic Scattering site when they capture a carriers.

Empirical & SPICE model

\[ S_{Id} = \frac{M g_m^2}{C_{ox}^2 W L} \frac{1}{f^\beta} \]

Since \[ S_{Id} = g_m^2 S_{Vg} \]

\[ S_{Vg} = \frac{M}{C_{ox}^2 W L} \frac{1}{f^\beta} \]

- \( S_{Id} \): PSD of drain current
- \( M \): empirical parameter
- \( g_m \): Transconductance
- \( W, L \): Width & length
- \( C_{ox} \): Oxide capacitance per unit area
- \( \beta \): close to 1 in wide frequency range and in any case varies in a narrow range between 0.8 and 1.2
McWhorter's model (1/f noise)

- Common used SPICE noise model equations

\[ S_{Id} = \frac{K F^* I_d^{AF}}{C_{ox} W L_{eff} f^{EF}} \]

\[ I_d \quad : \text{drain current} \]
\[ K F \quad : \text{flicker noise coefficient} \]
\[ A F \quad : \text{flicker noise exponent} \]
\[ E F \quad : \text{flicker noise frequency exponent} \]
\[ L_{eff} \quad : \text{Effective gate length} \]

: NOIMOD= 1, 4

At Strong inversion in the Linear Region

: For strong inversion, in the linear region at low drain voltages

\[ S_{Vg} = \frac{q^2 N_{ot} 1}{C_{ox}^2 W L f} \]

\[ N_{ot} \quad : \text{Density of oxide traps} \]

\[ N_{ot}[cm^{-2}] = \frac{k T N_t(E)}{\gamma} \]

\[ N_t(E)[cm^{-3}eV^{-1}] \]

\[ \gamma \quad : \text{McWhorter tunneling parameter} \]
McWhorther’s model (1/f noise)

\[ S_{ld} = g_m^2 S_{vg} = \frac{q^2}{C_{ox}} \left( \frac{I_d}{V_g - V_t} \right)^2 \frac{N_{ot}}{WL} \frac{1}{f} = \frac{q^2 \mu V_d I_d N_{ot}}{C_{ox} L^2 (V_g - V_t) f} \]

Corresponds to the SPICE model given by

\[ S_{ld} = \frac{K F * I_d^{AF}}{f^{EF} C_{ox} L_{eff}^2} \]

Assuming that \( K F \equiv \mu q^2 N_{ot} / (V_g - V_t) V_d \), \( AF = 1 \), \( EF = 1 \)

At Strong inversion in the Saturation Region

- In saturation, for \( V_d >> V_{ds, sat} \equiv (V_g - V_t) \)

\[ S_{vg} = \frac{q^2}{C_{ox}^2} \frac{N_{ot}}{WL} \frac{1}{f} \]

\[ S_{ld} \approx \frac{q^2 \mu N_{ot} I_d}{C_{ox} L_{eff}^2} \frac{1}{f} \]

Since in saturation \( g_m = \sqrt{2 C_{ox} \mu W I_d / L_{eff}} \)
McWhorter’s model (1/f noise)

\[ S_{Id} \approx \frac{q^2 \mu N_{ot} I_d}{C_{ox} L_{eff}^2} \frac{1}{f} \]

Corresponds to the SPICE model given by

\[ S_{Id} = \frac{K F * I_d^{AF}}{f^{EF} C_{ox} L_{eff}^2} \]

Assuming that \( K F \approx \mu q^2 N_{ot} \), \( AF = 1 \), \( EF = 1 \)

At Subthreshold Region

: At weak inversion below threshold

\[ S_{Vg} \equiv \frac{1}{C_{ox}^2} S_{Q_{inv}}(f) = \frac{1}{C_{ox}^2} \left[ \frac{C_{inv}}{C_{ox} + C_d + C_{inv}} \right]^2 q^2 N_{ot} \frac{1}{WL} \frac{1}{f} \]
McWhorter’s model (1/f noise)

\[ S_{vg} = \frac{1}{C_{ox}^2} S_{Q_{inv}}(f) = \frac{1}{C_{ox}^2} \left[ \frac{C_{inv}}{C_{ox} + C_d + C_{inv}} \right]^2 \frac{q^2 N_{ot}}{WL} \frac{1}{f} \]

Since at subthreshold \( C_{inv} \ll C_{ox} + C_d \)

Capacitance ratio between the oxide capacitance and the depletion capacitance is defined by

\[ n = \frac{C_{ox} + C_d}{C_{ox}} \]

Then, \( S_{vg} = \left( \frac{C_{inv}}{C_{ox}} \right)^2 \frac{1}{n^2} \frac{q^2 N_{ot}}{C_{ox}} WL \frac{1}{f} \)

\[ \text{Sv}_{vg} \text{ will be significantly reduced compared to that in saturation.} \]

At subthreshold, the drain current is related to gate voltage by

\[ I_d \equiv I_0 e^{qV_g/kT} \quad , \quad n = \frac{C_{ox} + C_d}{C_{ox}} \]

\[ \therefore g_m \equiv qI_D C_{ox} / kT(C_{ox} + C_d) \]
McWhorther’s model (1/f noise)

\[ S_{ld} = \frac{C_{inv}^2}{(C_{ox} + C_d)^4} \frac{q^4}{(kT)^2} I_d^2 \frac{N_{ot}}{WL} \frac{1}{f} = \left( \frac{C_{inv}}{C_{ox}} \right)^2 \frac{1}{n^4} \left( \frac{q}{kT} \right)^2 \frac{q^2 N_{ot}}{C_{ox}} \frac{I_d^2}{WL* f} \]

\( S_{ld} \) Increases with \( I_d^2 \)

Corresponds to the SPICE model given by

\[ S_{ld} = \frac{K F * I_d^{AF}}{C_{ox} W L_{eff}} \frac{1}{f^{EF}} \]

Assuming that

\[ K F = \left( \frac{C_{inv}^2}{C_{ox}^3} \right) \left( 1 / n^4 \right) \left( q / kT \right)^2 q^2 N_{ot} \quad AF = 2 \quad EF = 1 \]

: Voltage dependence \( N_{ot} \) is not considered.

KF had different dimensions (KF is measured [Amper*F] in saturation and linear regions and [F] in the subthreshold region.)
McWhorther’s model (1/f noise)

- Device Information: P-channel and n-channel MOS for analog applications (2 um technology)
  - 2um process: Nwell(2um), XJ: 0.2um, Tox:400A, field oxide:4000A, Vt:0.7V (nmos), -0.9V(pmos)
  - Subthreshold slope: 85mV/decade
McWhorter’s model (1/f noise)

- Device Information: P-channel and n-channel MOS for analog applications (0.5um technology) 0.5um process: Twin well, Tox: 115A, L_eff=0.4um, V_t: 0.55V(nmos), -0.65V (pmos)
- Tungsten silicide is formed over the polysilicon gate, subthreshold slope: 100mV/decade
McWhorter’s model (1/f noise)

In saturation regions, \( AF = 1 \)

\[
K_F = S_{ld} C_{ox} L_{eff}^2 f / I_d
\]

In subthreshold regions, \( AF = 1 \)

\[
K_F' = S_{ld} C_{ox} W L f / I_d^2
\]
Hooge’s model (1/f noise)

- The mobility fluctuation theory considers the flicker noise as a result of the fluctuation in bulk mobility based on Hooge’s empirical relation for the PSD of flicker noise.

\[
\frac{S_I}{I^2} = \frac{\alpha_H}{fN_{total}} \quad N_{total} : \text{Total number of carriers, } I : \text{mean current} \\
\alpha_H : \text{Hooge's empirical parameter} \quad 1 \times 10^{-7} \sim 1 \times 10^{-4}
\]

\[
\frac{1}{N} = \frac{q\mu R}{L^2} = \frac{q}{C_{ox}V_{G}WL}
\]

At below saturation (\(V_d < V_{dsat}, I_d < I_{dsat}\))

\[
\frac{S_I}{I^2} = \frac{\alpha_H q\mu R}{L^2 f} = \frac{\alpha_H q}{C_{ox}V_{gseff}WL_f} \propto \frac{1}{WL}
\]

Since \(I = (W/L)\mu C_{ox}V_{gseff}V_d\), \(R = V/I\)

\[
S_I = \frac{\alpha_H q\mu^2 C_{ox}V_{gseff}V_d^2W}{fL^3} \propto \frac{W}{L^3}
\]

\(V_{gseff}\) : Effective gate voltage

\(R\) : Channel resistance
Hooge's model (1/f noise)

- At saturation region

\[
\frac{S_{I_s}}{I_{sat}^2} = \frac{2\alpha_H}{fN_{total}}
\]

\[
S_{I_s} \equiv \frac{2\alpha_H I_{sat}^2}{N_f} = \frac{\alpha_H q \mu^2 C_{ox} V_{gseff}^3 W}{L^3 f} \propto \frac{W}{L^3}
\]

For \( S_{I_s} \) versus \( I_{sat} \) we then obtain

\[
S_{I_s} \equiv \frac{\sqrt{2\alpha_H q \mu^{1/2} I_{sat}^{3/2}}}{W^{1/2} C_{ox}^{1/2} L^{3/2} f}
\]

Since \( I_{sat} = \frac{(W/L) \mu C_{ox} V_{gseff}^2}{2} \)

\[
S_{I_s} \propto 1/W^{1/2} L^{3/2}
\]

Since \( S_{Vg} = S_{I_s} \cdot g_m^2 \)

\[
S_{Vg} \equiv \frac{\alpha_H q V_{gseff}}{2WLC_{ox}f} \propto \frac{1}{WL}
\]
Hooge’s model (1/f noise)

In the ohmic region \( V_{ds} < (V_{gs} - V_{th})/10 \)

\[
S_I = \frac{\alpha_H q \mu^2 C_{ox} V_{gseff} V_d^2 W}{fL^3} \propto \frac{W}{L^3}
\]

\( S_I \) is proportional to \( V_d^2 \)

At fixed drain and gate bias

\( S_I \propto W / L^3 \)

\( \alpha_H \) is inversely proportional to \( V_{gseff} \)

Since \( S_I \propto \alpha_H V_{gseff} \)

In saturation region \( \Delta N \) behavior is

\[
S_{Is} \propto \alpha_H V_{gseff}^3 \propto V_{gseff}^2
\]
Hooge’s model (1/f noise)

In the $\Delta N$ model we find for $\alpha_H$

$$\alpha_H = \frac{q x_o D_o kT (x_o / x_2)}{\varepsilon_o \varepsilon_r V_{gseff}} \frac{t_{ox}}{V_{gseff}} = \frac{\alpha_H E_c t_{ox}}{V_{gseff}} \propto \frac{t_{ox} T}{V_{gseff}}$$

$x_o$ : the characteristic decay length of the electron wave function ( ~ 1 A)

$x_o D_o$ : trap density per unit area and unit energy $\approx 10^{10} \text{cm}^{-2} (\text{eV})^{-1}$

$x_2$ : largest trapping distance ( ~ 30A)

$$\alpha_H = \frac{\pi m_e e^3}{16 \varepsilon_{av} h kT}$$ By Ning and Sah

$\varepsilon_{av} = (\varepsilon_{si} + \varepsilon_{ox}) / 2$, $m_e$ : electron effective mass

Hooge’s parameter extracted from the flicker noise versus gate voltage
### McWhorter & Hooge noise model (1/f noise)

- McWhorter & Hooge noise model (1/f noise)

<table>
<thead>
<tr>
<th></th>
<th>$\Delta n$</th>
<th>$\Delta \mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lin.</td>
<td>$S_I = \left( \frac{q}{C_{ox}} \right)^2 \frac{kTN_T(E_f)}{\gamma fWL} \left( \frac{I_d}{V_{GS} - V_T} \right)^2$</td>
<td>$S_I = \frac{\alpha q \mu^2 C_{ox} (V_{GS} - V_T) W_{DS}^2 W}{f L^3}$ $\propto \frac{W}{L^3}$</td>
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<tr>
<td></td>
<td>$S_{Vg} = \frac{S_I}{g_m^2} = \left( \frac{q}{C_{ox}} \right)^2 \frac{kTN_T(E_f)}{\gamma fWL}$</td>
<td>$\frac{S_I}{I_d^2} = \frac{\alpha q}{f C_{ox} (V_{GS} - V_T) W L}$ $\propto \frac{1}{WL}$</td>
</tr>
<tr>
<td>Sat.</td>
<td>$S_I = \frac{kTN_T(E_f) \mu q I_d}{\gamma f C_{ox} L^2}$</td>
<td>$S_I = \frac{\alpha q \mu^2 C_{ox} (V_{GS} - V_T)^3 W}{2 f L^3}$</td>
</tr>
<tr>
<td></td>
<td>$S_{Vg} = \left( \frac{q}{C_{ox}} \right)^2 \frac{kTN_T(E_f)}{\gamma fWL}$ $\propto \frac{1}{C_{ox}^2}$</td>
<td>$S_{Vg} = \frac{\alpha q C_{ox} (V_{GS} - V_T)}{2 W L C_{ox} f}$ $\propto \frac{1}{C_{ox}}$</td>
</tr>
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</table>
McWhorter & Hooge noise model (1/f noise)

- 1/f noise investigation of the 0.35um n and p type MOSFET
  - Device DC characteristics
    1. Length: 0.35um, Width: 200um (nmos), 264um (pmos)
    2. nmos $g_m$: 40mS/mm, pmos $g_m$: 11.4mS/mm
    3. nmos Mobility: 391cm$^2$/Vs, pmos Mobility: 96 cm$^2$/Vs
    4. Low frequency noise measured with HP35670A in the 1Hz~100kHz

- Reference paper: On-Wafer Low frequency noise investigation of the 0.35um n and p type Mopfets dependence upon the gate geometry

<table>
<thead>
<tr>
<th>n type</th>
<th>No.of fing.</th>
<th>w (μm)</th>
<th>p type</th>
<th>No.of fing.</th>
<th>w (μm)</th>
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<tbody>
<tr>
<td>11</td>
<td>10</td>
<td>10</td>
<td>21</td>
<td>16</td>
<td>16.5</td>
</tr>
<tr>
<td>41</td>
<td>16</td>
<td>12.5</td>
<td>24</td>
<td>20</td>
<td>13.5</td>
</tr>
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<td>44</td>
<td>20</td>
<td>10</td>
<td>27</td>
<td>20</td>
<td>13.5</td>
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<tr>
<td>47</td>
<td>32</td>
<td>6.25</td>
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<td>16</td>
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</tr>
<tr>
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<td>16</td>
<td>12.5</td>
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<td>57</td>
<td>32</td>
<td>6.25</td>
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Table1. gate size of n type and p type MOSFETs
McWhorther & Hooge noise model (1/f noise)

Figure 1. Transconductance $g_m$ and drain current $I_D$ of $n$ and $p$ type MOSFETs biased with $V_D=0.1$ V. (solid line is device 41, dashed-dotted line is 47, dashed line is 31 and dotted line is 37), a).

$SI_D$ for the $n$ type device 44. (1. is $I_D=7.5\times10^{-11}$ A, 2. is $I_D=10\times10^{-11}$ A, 3. $I_D=91\times10^{-11}$ A, 4. $I_D=0.6\times10^{-8}$ A, 5. $I_D=7.18\times10^{-8}$ A, 6. $I_D=5.95\times10^{-7}$ A, 7. $I_D=2.3\times10^{-6}$ A, 8. $I_D=1.8\times10^{-5}$ A, 9. $I_D=6.79\times10^{-5}$ A, 10. is $I_D=2.47\times10^{-4}$ A, 11. $I_D=7.38\times10^{-4}$ A, 12. $I_D=10.07\times10^{-4}$ A), b).
McWhorther & Hooge noise model (1/f noise)

Figure 2. a) $S(I_d)$ at 10 Hz for the devices 41 (squares) and 44 (blue circles) lines.
   b) $S(I_d)/I_d^2$ for 41 (squares). Solid line is simulated and 44 (circle, red dashed line is simulated.)
Figure 3. Sid/Id² versus drain current for the 34 (circle are measured, solid black line is simulated and dotted red line is mobility fluctuation and dashed line is trapping related noise.

Table 2. Parameter extracted from low frequency noise Analysis

<table>
<thead>
<tr>
<th>Device</th>
<th>N' \text{lo} (\text{eV}^{-1} \text{cm}^{3})</th>
<th>\alpha_f</th>
<th>S_{VfB} (\text{V}^2/\text{Hz})</th>
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</thead>
<tbody>
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<td>N type</td>
<td>11</td>
<td>9.7x10^{16}</td>
<td>0.6x10^{-7}</td>
</tr>
<tr>
<td>P type</td>
<td>27</td>
<td>6.9x10^{15}</td>
<td>0.2x10^{-7}</td>
</tr>
<tr>
<td>P type</td>
<td>34</td>
<td>1.96x10^{16}</td>
<td>0.8x10^{-7}</td>
</tr>
<tr>
<td>N type</td>
<td>41</td>
<td>6.9x10^{15}</td>
<td>0.6x10^{-7}</td>
</tr>
<tr>
<td>N type</td>
<td>44</td>
<td>1.0x10^{17}</td>
<td>0.2x10^{-7}</td>
</tr>
<tr>
<td>N type</td>
<td>51</td>
<td>3.5x10^{17}</td>
<td>0.6x10^{-7}</td>
</tr>
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</table>
McWhorther & Hooge noise model (1/f noise)

- Room temperature 1/f noise behaviour for NMOS and PMOS Device information: 0.5um technology, Tox: 485A, W=12um, L=3um (Nmos)

Reference paper: Flicker noise in cmos transistors from subthreshold to strong inversion at various temperature.

Fig 1. in linear region
Fig 2. in saturation region

$$S_{Vg} = \frac{q^2}{C^2_{ox}} \frac{N_{ot}}{WL} \frac{1}{f^n}$$

$$N_{ot}[cm^{-2}] = \frac{kT N_t(E)}{\gamma}$$

No gate bias dependence!!

$$\gamma = 10^8 cm^{-3}$$

$$N_t kT = 2 \times 10^{15} \sim 2 \times 10^{16}$$

Input referred noise spectra in these n channel TR vary very little as the gate voltage changes, both in the linear and saturation Regions of operation. The “independence” from gate bias voltage in the input referred noise suggests that flicker noise from these n-channel devices is due to carrier-density fluctuation rather than mobility fluctuation.

LDD structure: short channel LDD n type devices, strong gate bias dependence was observed. The gate bias dependent component of noise by attributing it to the voltage dependent series resistance of the LDD structure at the drain end of the device.
McWhorther & Hooge noise model (1/f noise)

- Device information: 0.5um technology, Tox: 485A, W=12um, L=4um (Pmos)

\[ S_{I} \frac{I^{2}}{fN_{total}} = \frac{\alpha_{H}}{fN_{total}} \]

\[ S_{VG}(f) = \left( \frac{q}{C_{ox}} \right) \frac{\alpha_{H}}{WLf} (V_{GS} - V_{T}) \]

: In the linear regions

gate bias dependence

It very often shows gate voltage dependence in both the linear and saturation regions of operations. Input referred power in p channel devices can be 10~100 times less as compared to n channel transistors. This noise is for mobility fluctuation. This gate bias dependence has been explained by buried channel conduction in ion-implanted devices, where bulk mobility fluctuation noise dominate.
McWhorter & Hooge noise model (1/f noise)

The noise spectra shows an increase in slope at lower frequencies at very low temperatures. It probably due to a generation–recombination noise source at low frequency. The flicker noise of nmos at low temperature does not decrease in any significant order of magnitude!!

PMOS device
The noise power decreases as the temperature decreases to about 150K and the slope of the spectrum shows no change. However, noise increases when the temperature is lowered beyond 150K. The slope of the spectrum becomes very small.
McWhorter & Hooge noise model (1/f noise)

- In the subthreshold region operation

**NMOS device**
It can be seen that input referred noise in the subthreshold region has the same behavior as that in the strong inversion. No gate bias dependence is observed.

**PMOS device**
Input referred noise in pmos, the input referred noise decreases in magnitude as the device bias is varied from subthreshold into Strong inversion.

---

**Fig 8.** W=100um/ L=10um nmos, Vg changes from subthreshold to strong inversion

**Fig 9.** W=100um/L=5um pmos, Vg varies from subthreshold to strong inversion
BSIM3 1/f Noise Concept

- BSIM3 Noise model concept
  1. Incorporates both the oxide-tap-induced carrier number and correlated surface mobility fluctuation mechanisms.
  2. The model is applicable to long channel, as well as submicron n and p channel MOSFETs.
  3. Noise characteristics over the linear, saturation, and subthreshold operating regions.

Fraction of change of the channel current

\[
\frac{\delta I_d}{I_d} = - \left[ \frac{\delta \Delta N}{\Delta N} \pm \frac{\delta \mu}{\mu} \right]
\]

\[
= - \left[ \frac{1}{\Delta N} \frac{\delta \Delta N}{\delta \Delta N_t} \pm \frac{1}{\mu} \frac{\delta \mu}{\delta \Delta N} \right] \delta \Delta N_t \quad \text{- equ 1}
\]

First term: carrier number of fluctuation
Second term: fluctuation of surface mobility

\[N : \text{Carrier density}\]
\[N_t : \text{the number of filled traps per unit area}\]
**BSIM3 1/f Noise Concept**

\[ \Delta N = NW \Delta y \quad \text{- equ 2} \quad \Delta N_t = N_t W \Delta y \quad \text{- equ 3} \]

The ratio of the fluctuation in carrier number to fluctuations in occupied trap number is close to unity at strong inversion.

A general expression for \( R \) is

\[ R = \frac{\delta \Delta N}{\delta \Delta N_t} = -\frac{C_{inv}}{C_{ox} + C_{inv} + C_{it} + C_{dep}} \quad \text{- equ 4} \quad C_{inv} \approx \frac{q^2}{kT} \quad \text{- equ 5} \]

More concise form as

\[ R = -\frac{N}{N + N^*} \quad \text{- equ 6} \quad \text{where} \quad N^* = \frac{kT}{q^2} (C_{ox} + C_d + C_{it}) \]

Typical values of \( N^* \) is \( 1 \sim 5 \times 10^{10} \text{ cm}^{-2} \)

To evaluate \( \frac{\delta \mu_{\text{eff}}}{\delta \Delta N_t} \)

\[ \frac{1}{\mu_{\text{eff}}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ox}} = \frac{1}{\mu_n} + \alpha N_t \quad \text{- equ 7} \quad : \text{Matthiessen’s rule} \]

\( \mu_{ox} \) : is mobility limited by oxide charge scattering, \( \alpha \) : Scattering coefficient
BSIM3 1/f Noise Concept

\[
\frac{1}{\mu_{\text{eff}}} = \frac{1}{\mu_n} + \frac{1}{\mu_{\text{ox}}} = \frac{1}{\mu_n} + \alpha N_t \implies \frac{\delta \mu}{\delta \Delta N_t} = -\frac{\alpha \mu^2}{W \Delta y} \quad \text{- equ 8}
\]

Substituting equ 4 and equ 8 into equ 1 yields

\[
\frac{\delta I_d}{I_d} = -\left( \frac{R}{N} \pm \alpha \mu_{\text{eff}} \right) \frac{\delta \Delta N_t}{W \Delta y} \quad \text{- equ 9}
\]

Therefore, the power spectral density of the local current fluctuations can be written as:

\[
S_{\Delta I_d} = \left( \frac{I_d}{W \Delta y} \right)^2 \left( \frac{R}{N} + \alpha \mu_{\text{eff}} \right)^2 S_{\Delta N_t} \quad \text{- equ 10}
\]

\[
S_{\Delta N_t} = \int_{E_v}^{E_c} \int_0^W \int_0^{\text{Tox}} 4N_t(E)\Delta x f_t(1 - f_t) \frac{\tau}{1 + \omega^2 \tau^2} dxdydz = \frac{N_t(E_{Fn})kTW\Delta y}{f \gamma} \quad \text{- equ 11}
\]

\( \gamma \) : Attenuation coefficient of the electron wave function in the oxide

\( \tau \) : Trapping time constant, \( f_t = [1 + \exp(E - E_{Fn})/kT]^{-1} \) : trap occupied function

\( E_{Fn} \) : electron quasi Fermi level, \( \omega = 2\pi f \)
Substituting eqn 11 into eqn 10 yields

$$S_{\Delta I_d} = \frac{kTI_d^2}{\gamma \star f \star W \star \Delta y} \left( \frac{R}{N} + \alpha \mu_{\text{eff}} \right)^2 N_t$$ - eqn 12

Total drain current noise power is then:

$$S_{I_d} = \frac{1}{L^2} \int_0^L S_{\Delta I_d} \Delta y dy = \frac{1}{L^2} \int_0^L \frac{kTI_d^2}{\gamma \star f \star W} \left( \frac{R}{N} + \alpha \mu_{\text{eff}} \right)^2 N_t(E_{F_n}) dy$$

$$= \frac{kTqI_d\mu_{\text{eff}}}{\gamma \star f \star L^2} \int_0^V N_t(E_{F_n})(1 \pm \alpha \mu_{\text{eff}}N R^{-1})^2 \frac{R^2}{N} dV$$ - eqn 13

It can be rewritten as

$$S_{I_d} = \frac{kTqI_d\mu_{\text{eff}}}{\gamma \star f \star L^2} \left( \int_0^V N_{t^*}(E_{F_n}) \frac{R^2}{N} dV \right)$$ - eqn 14

With

$$N_{t^*}(E_{F_n}) = N_t(E_{F_n})(1 \pm \alpha \mu_{\text{eff}}N R^{-1})^2$$
BSIM3 1/f Noise Concept

Let \[ N_t^* = N_t(E_{Fn})(1 \pm \frac{\alpha \mu_{eff} N}{R})^2 = A + BN + CN^2 \] - equ 15

\[ A = N_t, \quad B = \pm \frac{2 \alpha \mu_{eff} N_t}{R} \]

In the linear region \( V_d \ll V_{d,sat} \)

Using eqn 13 and Id equation as following

\[ I_d = Wv(y)qN(y) \quad qN(y) = C_{ox}(V_{gs} - V_{th}(y) - V(y)) \]

\[ qN(y) = C_{ox}(V_{gs} - V_{th}(y) - aV(y)) \] - eqn 16  \( a: \) takes into account bulk charge effect

\[ a = 1 + \frac{1}{2} gK(\psi_s - V_b)^{-1/2} \]

\[ g = 1 - \frac{1}{1.744 + 0.8364(\psi_s - V_b)} \]

By substituting eqn 16 into eqn 14

\[ S_{Id} = \frac{kTq^2 I_d \mu_{eff}}{a \gamma f L C_{ox}} \left( \int_{N_0}^{N_L} N_t^*(E_{Fn}) \frac{R^2}{N} dV \right) \] - eqn 17

with

\[ qN_o = qN(0) = C_{ox}(V_{gs} - V_{th}) \]

\[ qN_L = qN(L) = C_{ox}(V_{gs} - V_{th} - aV_d) \]
BSIM3 1/f Noise Concept

\[ R = -\frac{N}{N + N^*} \]
\[ N^* = \left(\frac{kT}{q^2}\right)(C_{ox} + C_d + C_{it}) \] - eqn 18

Substituting above equation into equ 17 and performing the integration yield

\[ S_{ld} = \frac{q^2kTI_d\mu_{eff}}{\alpha\gamma L^2C_{ox}} \left[ A\ln\frac{N_0 + N^*}{N_L + N^*} + B(N_0 - N_L) + \frac{C}{2}(N_0^2 - N_L^2) \right] \] - eqn 19

\[ \rightarrow \text{Linear region equation} \]

In the saturation region \( V_d >> V_{d,sat} \)

At \( V_d > V_{d,sat} \), the channel current can be divided into the “triode” and “pinch-off” regions

Accordingly, the flicker noise power is made up of two parts:

\[ S_{ld} = \frac{kTI_d^2}{\gamma L^2W} \left( \int_{-L}^{L} N^*_l(E_{Fn}) \frac{R^2}{N(y)^2} dy + \int_{L-\Delta L}^{L} N^*_l(E_{Fn}) \frac{R^2}{N(y)^2} dy \right) \] - eqn 20

\[ S_{ld} = \frac{q^2kTI_d\mu_{eff}}{\alpha\gamma L^2C_{ox}} \left[ A\ln\frac{N_0 + N^*}{N_L + N^*} + B(N_0 - N_L) + \frac{C}{2}(N_0^2 - N_L^2) \right] + \frac{kTI_d^2\Delta L}{\gamma^* f^* L^2*W} \left[ A + BN_L + CN_L^2 \right] \] - equ 21

with \( qN_o = qN(0) = C_{ox}(V_{gs} - V_{th}) \), \( qN_L = qN(L) = C_{ox}(V_{gs} - V_{th} - aV_d) \) Saturation region equation
BSIM3 1/f Noise Concept

In the subthreshold region, diffusion current dominates, and therefore the drain current diminished exponentially with decreasing gate voltage

$$qN(V) = \frac{kT}{q} C_d^* \exp \left[ \frac{1}{n} \left( \frac{kT}{q} (V_g - V_g^*) - \frac{kT}{2} \frac{\psi_F}{q} - \frac{kT}{q} V \right) \right] \quad \text{- eqn 22}$$

with \( n = (C_{ox}^* + C_d^* + C_{it}^*) / C_{ox} \)

Substituting eqn 22 into eqn 14 and after some manipulation yields

$$S_{ld} = \frac{k^2 T^2 I_{st} \mu_0}{\gamma \ell^2} \int_{N_L}^{N_0} \frac{N_t^* (E_{Fn})}{(N^* + N)^2} dN \quad \text{- eqn 23} \quad \text{where} \quad qN_o = \frac{qK_s}{kT} \exp \left[ \frac{1}{n} \left( \frac{kT}{q} (V_g - V_{th}) \right) \right]$$

$$qN_L = qN_o [1 - \exp(-\frac{qV_d}{kT})] \quad I_{st} = \frac{W}{L} \mu_0 \int_0^{V_d} qN(V) dV = \frac{W}{L} \mu_0 C_d^* \left( \frac{q}{kT} \right)^2 \exp \left[ \frac{1}{n} \left( \frac{kT}{q} (V_g - V_g^*) - \frac{1}{2} \frac{kT}{q} \psi_F \right) \right] \left( 1 - e^{-\frac{qV_d}{kT}} \right)$$

In the subthreshold region it is reasonable to assume that \( N << N^* \) and

$$N_t^* (E_{Fn}) = A + BN + CN^2 \approx A$$

Then eqn 17 turns out to be

$$S_{ld} = \frac{Ak T I_{st}^2}{\gamma \ell W L N^*^2} \quad \rightarrow \text{Subthreshold region equation}$$
BSIM3 1/f Noise Concept

Comparison measure and simulation
Device information: 3um CMOS technology, W=9.5um, L=4.5um, Tox=50nm, Nsub: 1X10^{15} cm^{-3}
Reference paper: Physical based mosfet noise model for circuit simulators

The noise spectrum clearly reveals a very close to unity. The observed frequency dependence a uniform Spatial distribution near the interface, as a non-uniform distribution will cause to deviate from unity!!

But most of experimental values for the slope of noise Spectra density are rarely exactly 1 but varies from 0.7 to 1.2. This might be due to a number of reasons, Such as generation-recombination noise and non-uniform distribution of traps.
BSIM3 1/f Noise Concept

The measured drain current noise power at 100Hz

1. The input referred noise power is equal to the drain current noise power divided by the square of the transconductance (gm²).

2. The input referred noise is almost independent of the bias point in both linear and saturation regions.
BSIM3 1/f Noise Concept

- Another n channel MOSFET by submicron NMOS technology
  Device information: $\text{Tox} = 8.6\text{nm}$, $N_{\text{sub}} = 5 \times 10^{17} \text{cm}^{-3}$, $W=4.5\text{um}$, $L=4.5\text{um}$

![Fig3. bias dependence of the drain current noise power](image1)

![Fig4. Input referred noise power (S_v)](image2)

The input referred noise power of the submicron technology shows strong dependence on the bias point in both linear and saturation regions.
BSIM3 1/f Noise Concept

- Another n channel MOSFET by submicron NMOS technology
  Device information: $\text{Tox} : 28.5\text{nm}$, $\text{Nsub} : 2.6 \times 10^{16} \text{ cm}^{-3}$, $W=20\text{um}$, $L=1.9\text{um}$

The input referred noise power of the submicron technology shows strong dependence on the bias point in both linear and saturation regions.

Fig 5. Noise power measure in strong inversion, as well as subthreshold regions for N channel MOSFET

Fig 6. Bias dependence of noise power in the subthreshold and strong inversion regions
BSIM3 1/f Noise Concept

- Another n channel MOSFET by submicron NMOS technology
  Device information: Tox: 8.6nm, Nsub: 5 \times 10^{17} \text{ cm}^{-3}, \text{W}=20\text{um}, \text{L}=0.65\text{um}

1. The short channel effects on the flicker noise characteristics are evident through comparison of Fig 6 and 8.
2. For short channel device, the drain current noise power continues to increase with the drain voltage beyond the saturation point in both the strong inversion and subthreshold regions.
BSIM3 1/f Noise Concept

- Another p channel MOSFET by submicron PMOS technology
  Device information: \( \text{Tox} : 8.8\text{nm}, \text{Nsub} : 1\times10^{14} \text{ cm}^{-3} \), \( W=4\text{um}, L=5\text{um} \)

Fig 9. noise power measure in strong inversion, as well as subthreshold regions for P channel MOSFET

Fig 10. Bias dependence of noise power in the subthreshold and strong inversion regions
BSIM3 1/f Noise Concept

- Another p channel MOSFET by submicron PMOS technology
  Device information: \( \text{Tox} : 8.8\text{nm}, \text{Nsub} : 1 \times 10^{14} \text{ cm}^{-3}, \text{W} = 3.2\text{um}, \text{L} = 2\text{um} \)

Generation – recombination symptom

Significant deviation from the 1/f frequency dependence.

The additional noise source is believed to be the g-r noise arising from the substrate defect centers, which were introduced during boron implantation.

Fig 7. bias dependence of the drain current noise power of a buried channel p channel MOSFET
Impact of process scaling on 1/f noise

- The influence of the gate-oxide thickness, substrate dope, and the gate bias on the input-referred spectral 1/f noise density
  Reference paper: Impact of process scaling on 1/f noise in advanced CMOS technologies.

Device information: $W=10\,\mu m$, $L=4\,\mu m$ (Nmos, Pmos), $Tox: 2$, 3.6, 5, 7.5, 10, and 20nm
Na variants of $5 \times 10^{17} \, cm^{-3}$ and $5 \times 10^{16} \, cm^{-3}$
Average $S_{Vg}$ at 100Hz

Fig 1. drain current spectral density vs frequency with the identical TOX, dope concentration Na, and identical bias conditions (PMOS)

Fig 2. Interface trap density $N_{it}$ versus $Tox$
Impact of process scaling on 1/f noise

Fig 3. $S_{Vg}$ versus $Tox$ (NMOS)

Fig 4. $S_{Vg}$ versus $Tox$ (PMOS)

$S_{Vg}$ decreases with decreasing $Tox$. Fig 5 shows that $S_{Vg}$ of NMOS depends stronger on $Tox$ than that of PMOS.

Fig 5. The power $p$ versus $Vgt$
For Large Tox, $S_{Vg}$ of PMOS shows a stronger dependence on Vgt than that of NMOS. For small Tox, both NMOS and PMOS show a strong Vgt dependence. The substrate doping concentration Na affects $S_{Vg}$ as well. With a 10X increase of Na, it enlarges with a factor 3+/- 1.5.
How to modeling for SPICE2 1/f Noise

Reference: 1/f noise modeling for semiconductors (F. Sischka, Agilent Technologies)

\[ S_{Id} = \frac{KF \cdot I_d^{AF}}{f^{EF} C_{ox} L_{eff}^2} \quad \text{: Drain current noise spectral density} \]

\[ \overline{i_{nD}^2} = \frac{KF \cdot I_d^{AF}}{f^{EF} C_{ox} L_{eff}^2} \Delta f \quad \text{: Drain – source effective noise current} \]

with \[ C_{ox} = \frac{\varepsilon_o \varepsilon_{si}}{T_{ox}} = \frac{3.45E-11}{T_{ox}} \]

\[ L_{eff} = L_{drawn} - 2 \left( L_{INT} + \frac{LL}{L_{LN}} + \frac{LW}{W_{WN}} + \frac{LWL}{L_{LN} \cdot W_{WN}} \right) \]

Or simplified: \[ L_{eff} = L_{drawn} - 2L_{INT} \]
How to modeling for SPICE2 (1/f Noise)

Normalize to $\Delta f$ then set $\Delta f = 1\text{Hz}$

$$S_{ld} = \frac{i_{nD}^2}{1\text{Hz}} = \frac{KF * I_d^{AF}}{f^{EF} C_{ox} L_{eff}^2} \quad \Rightarrow \text{Eqn 1}$$

Step 1: EF parameter extraction (1/f slope) : A log conversion of eqn 1

$$\log_{10} S_{ld} = \left( KF * I_d^{AF} \right) / C_{ox} L_{eff}^2 - EF * \log_{10} f$$

Constant

We apply a regression curve fitting ..The parameter EF is the –slope

Step 2: EF slope is now modeled , we can get rid of it by multiplying the measured curve with the frequency point $f^{EF}$

$$S_{ld} f^{EF} = \frac{KF * I_d^{AF}}{C_{ox} L_{eff}^2} \quad \Rightarrow \text{Eqn 2}$$
How to modeling for SPICE2 (1/f Noise)

\[ S_{ld@1Hz} = \frac{KF \times I_d^{AF}}{C_{ox}L_{eff}^2} \quad \text{: identify the value of the } 1/f^E \text{ noise at } 1\text{Hz} \]

\[ \rightarrow \text{Eqn 3} \]

A log conversion of eqn 3

\[ \log_{10}(S_{ld@1Hz}) = \log_{10}(\frac{KF}{C_{ox}L_{eff}^2}) + AF \times \log_{10}(I_d) \]

What can be interpreted as a linear function like

\[ y = bx + a \]

where

\[ y = \log_{10}(S_{ld@1Hz}) \]

\[ a = \log_{10}\left(\frac{KF}{C_{ox}L_{eff}^2}\right) \]

\[ b = AF \quad x = \log_{10}I_d \]

The noise parameters AF and KF are then calculated after

\[ AF = b \quad KF = C_{ox}L_{eff}^2 \times 10^a \]
How to modeling for SPICE2 (1/f Noise)

Fig 1. $V_g = 0.6\,\text{V}, \, V_{ds} = 1\,\text{V}$

Fig 2. $V_g =$ sweep, $V_{ds} = 1\,\text{V}$
How to modeling for SPICE2 (1/f Noise)

Fig 3. $V_g = 0.6V$, $V_{ds} = 1V$

$\text{Si}_d (A^2/Hz)$

$\text{EF} \text{ parameter extraction}$

Fig 4. $V_g = \text{sweep}$, $V_{ds} = 1V$

Multiply by $f^{\text{EF}}$ in order to easier

Extract the 1Hz value of the noise
How to modeling for SPICE2 (1/f Noise)

Fig 5 . Noise spectra density @ 1Hz

Fig 6 . Noise spectra density @ 1Hz versus Id_current

AF, KF parameters extraction
How to modeling for SPICE2 (1/f Noise)

Fig 7. Noise spectra density versus Frequency
How to modeling for BSIM3V3 (1/f Noise)

- MOSFET investigated in all operating regions. → By Heijningen et al (linear and saturation range in strong inversion and subthreshold)

Reference Paper: CMOS 1/f noise modeling and extraction of BSIM3 parameters using a new extraction procedure.

1) In the subthreshold region

\[
S_{wi}(f) = \frac{NOIA \times kT \times I_d^2}{q W_{eff} L_{eff} f^E F N^*} = \frac{NOIA \times kT \times I_d^2}{q W_{eff} L_{eff} f^E F (4 \times 10^{36})} : \text{BSIM3 V3}
\]

NOIA is the subthreshold noise parameter

\[
N^* = \frac{kT}{q} (C_{ox} + C_d + C_{it})
\]
How to modeling for BSIM3V3 (1/f Noise)

To ensure the continuity between subthreshold and above threshold data:

\[ S_{Id}(f) = \frac{S_{wi}(f)S_{lim}(f)}{S_{wi}(f) + S_{lim}(f)} \quad \text{: Linking method} \quad \Rightarrow \text{Eqn 2} \]

Where \( S_{lim}(f) \) is the flicker noise measured at \( V_{GS} = V_T + 0.1 \)

2) In the above threshold region:

In the strong inversion ( \( V_{GS} \geq V_T + 0.1 \)):

\[ S_{Id} = \frac{q^2kT_d \mu_{eff}}{\alpha g f^2 C_{ox}} \left[ A \ln \frac{N_0 + N^*}{N_L + N^*} + B(N_0 - N_L) + \frac{C}{2}(N_0^2 - N_L^2) \right] + \frac{kT^2_d \Delta L}{\gamma * f * L^2 * W} \left[ A + B N_L + C N_L^2 \right] \]

\[ S_{Id} = \frac{q kT_d \mu_{eff}}{L_{eff}^2 C_{ox} f_{EF}} \left[ NOIA \ln \frac{N_0 + N^*}{N_L + N^*} + NOIB(N_0 - N_L) + \frac{NOIC}{2}(N_0^2 - N_L^2) \right] + \Delta L_{clm} \frac{kT^2_d}{q W_{eff} L_{eff}^2 f_{EF}^{EF}} \left[ NOIA + NOIB N_L + NOIC N_L^2 \right] \quad \text{: BSIM3 V3} \quad \Rightarrow \text{Eqn 3} \]
How to modeling for BSIM3V3 (1/f Noise)

\[
S_{Id} = \frac{qkTl_d \mu_{eff}}{L_{eff}^2 C_{ox} f_{EF}^{EF}} \left[ NOIA \ln \frac{N_0 + N^*}{N_L + N^*} + NOIB(N_0 - N_L) + \frac{NOIC}{2} (N_0^2 - N_L^2) \right] \\
+ \Delta L_{clm} \frac{kTl_d^2}{qW_{eff} L_{eff}^2 f_{EF}^{EF}} \left[ NOIA + NOIB N_L + NOIC N_L^2 \right] \rightarrow \text{BSIM3 V3) Eqn 3)}
\]

Since:

\[ \Delta n \quad \text{Model, saturation} \quad S_{Id} \equiv \frac{kTN_T(E_{Fn}) \mu_{eff} q^2 l_d}{\gamma * f} C_{ox} L_{eff}^2 \approx \frac{KF \cdot I_d^{AF}}{f^{EF} C_{ox} L_{eff}^2} \]

With:

\[ qN_0 = C_{ox} (V_{gs} - V_{th}) \quad , qN_L = C_{ox} (V_{gs} - V_{th} - \min(V_{DS}, V_{DSsat})) \]

\[ \Delta L_{clm} \quad \text{is the reduction in the electrical channel length due to the drain depletion into the channel in saturation regime.} \]

\[ \Delta L_{clm} = \text{Litl} \cdot \log \left( \frac{V_{DS} - V_{DSsat}}{\text{Litl} \cdot E_{at} + \frac{E_M}{E_{sat}}} \right) \quad E_{sat} \quad \text{: Corresponds to the critical electrical field at which the carrier velocity become saturated} \]

\[ \text{Litl} = \sqrt{\frac{\varepsilon_s T_{ox} X J}{\varepsilon_{ox}}} \]
How to modeling for BSIM3V3 (1/f Noise)

\[ E_{sat} = \frac{2V_{sat}}{\mu_0} \quad , V_{sat} = 1.5 \times 10^5 \text{ m/s} \]

\[ E_M \text{ is the maximum electric field } = 4.1 \times 10^7 \text{ V/m} \]

3) In the ohmic region (At Lower Vds biases)

\[ V_{ds} < \frac{(V_{gs} - V_{th})}{10} \]

The equation simplified ( Linear Equation )

\[ I_d = \mu_{eff} C_{ox} \frac{W_{eff}}{L_{eff}} (V_{GS} - V_T) V_{DS} \]

Then the expression “Eqn 3” can be approximated

\[ S_{Id} = \frac{kT \mu_{eff}^2 W_{eff}}{L_{eff}^3 \mu_{EF}} C_{ox} V_{DS}^2 \left( NOIB(V_{GS} - V_T) + \frac{C_{ox}}{q} NOIC(V_{GS} - V_T)^2 \right) \]

\[ \rightarrow \text{ Eqn 4} \]
How to modeling for BSIM3V3 (1/f Noise)

- Model Parameter Extraction

Step 1: from noise measurements performed in the subthreshold range, the parameter NOIA can be extracted using following equation.

\[
S_{wi}(f) = \frac{NOIA \cdot kT \cdot I_d^2}{qW_{eff}L_{eff}f^{EF}N^2} = \frac{NOIA \cdot kT \cdot I_d^2}{qW_{eff}L_{eff}f^{EF}(4 \times 10^{36})} : \text{BSIM3 V3}\]

A log conversion of eqn 1

\[
\log_{10}(S_{wi}(f)) = \log_{10}\left(\frac{NOIA \cdot kT}{qW_{eff}L_{eff}f^{EF}(4 \times 10^{36})}\right) + 2 \log_{10} I_d
\]

\[
y = bx + a
\]

\[
y = \log_{10}(S_{iw@1Hz})
\]
How to modeling for BSIM3V3 (1/f Noise)

\[ y = bx + a \]  
\[ y = \log_{10}(S_{iw@1Hz}) \]

\[ a = \log_{10}\left(\frac{NOIA \times kT}{qW_{eff}L_{eff} (4 \times 10^{36})}\right) = \log_{10}\left(\frac{NOIA \times 0.0259}{W_{eff}L_{eff} (4 \times 10^{36})}\right) \]

\[ \therefore NOIA = \frac{10^{a}W_{eff}L_{eff} \times (4 \times 10^{36})}{0.0259} \]

\[ bx = 2 \log_{10} I_d \]

**Step 2:** noise measurement are performed for various effective gate bias (Vgs-Vt) in the ohmic range (Typically Vds=50mV or 100mV). Then we obtained \( S_{ld}(f)/\mu_{eff}^2 \) vs Vgs-Vt, the obtained variations at low effective gate bias allow us to extract the NOIB parameter, So knowing NOIB, the parameter NOIC can be induced from the variation at large Vgs-Vt values.
How to modeling for BSIM3V3 (1/f Noise)

Step 3: three noise parameters will be matched with the help of noise measurements performed at higher Vds biases but always smaller than Vds,sat, in fact in this case the noise is a function of the three noise parameters and $\Delta L_{clm}$ remains equal to zero.

Step 4: in the saturation range, Litt and $\Delta L_{clm}$ are calculated if the junction depth is known, otherwise they deduced by a fit of the experimental data.

- Experimental detail

Device information: N type and P type transistors with various gate geometries W=20um, 0.8um ≤ L ≤ 20um, Tox: 16nm (0.8um CMOS technology)

Conductance parameters $(V_T, \mu_0, \theta, \Delta L, \Delta W, R_{acc})$ have been carried-out with a set of transfer characteristics $I_d(Vgs)$ collected in the ohmic range.
How to modeling for BSIM3V3 (1/f Noise)

Table 1. conductance parameters for n- and p-channel transistors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>n-channel</th>
<th>p-channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_T$ (V)</td>
<td>0.62</td>
<td>-0.62</td>
</tr>
<tr>
<td>$\theta$ (V$^{-1}$)</td>
<td>0.073</td>
<td>0.153</td>
</tr>
<tr>
<td>$\Delta L$ ($\mu$m)</td>
<td>0.3</td>
<td>-0.08</td>
</tr>
<tr>
<td>$\Delta W$ ($\mu$m)</td>
<td>0.45</td>
<td>0.71</td>
</tr>
<tr>
<td>$\mu_0$ (cm$^2$/Vs)</td>
<td>535</td>
<td>176</td>
</tr>
<tr>
<td>$R_{acc}$ ($\Omega$)</td>
<td>64</td>
<td>52</td>
</tr>
</tbody>
</table>

(for $W = 20\mu$m)
How to modeling for BSIM3V3 (1/f Noise)

For transistors with large area, straightforward 1/f noise have been observed and then EF=1

\[ y = bx + a \]
\[ y = \log_{10}(S_{iw@1Hz}) \]
\[ a = \log_{10}\left(\frac{NOIA \times 0.0259}{W_{eff}L_{eff}(4 \times 10^{36})}\right) \]
\[ NOIA = \frac{10^a W_{eff} L_{eff} \times (4 \times 10^{36})}{0.0259} \]
\[ bx = 2 \log_{10} I_d \]

Can be obtained taking into account S swing parameter of the subthreshold

\[ NOIA = 1.7 \times 10^{12} V^{-1} m^{-2} (PMOS) \]
\[ NOIA = 1.6 \times 10^{13} V^{-1} m^{-2} (NMOS) \]

Fig 1. Typical subthreshold Sid Versus drain current Ids at f=10Hz
How to modeling for BSIM3V3 (1/f Noise)

\[ N^* = (kT / q^2)(C_{ox} + C_d + C_{it}) \]

\[ S.S = \left( \frac{\partial \log I_d}{\partial V_{GS}} \right)^{-1} = 2.3 \frac{kT}{q} \ln \left( \frac{C_{ox} + C_d + C_{it}}{C_{ox}} \right) \]

\[ N^* = 4.2 \times 10^{14} \, m^{-2} \text{ For PMOS} \quad N^* = 5 \times 10^{14} \, m^{-2} \text{ For NMOS} \]

In the ohmic regions

\[ S_{ld} = \frac{kT \mu_{eff}^2 W_{eff}}{L_{eff}^3 f_{EF}^2} C_{ox} V_{DS}^2 \left( NOIB(V_{GS} - V_T) + \frac{C_{ox}}{q} NOIC(V_{GS} - V_T)^2 \right) \]

The parameter of NOIB is slope !!

1) For p type, it is proportional to \( V_{gs} - V_t \) as expected above equation .

2) For n type, it is independent of the effective gate voltage.

Fig 2. \( S_{d} / \mu_{eff}^2 \) in the ohmic range versus the effective gate voltage at \( f=1Hz \)
How to modeling for BSIM3V3 (1/f Noise)

At higher effective gate voltage (\( V_{gs-Vt} > 2V \)) a quadratic dependence is obtained:

\[
S_{id} = \frac{kT \mu_{eff}^2 W_{eff}}{L_{eff} f_{EF}^{1/2}} C_{ox} V_{DS}^2 \left( NOIB(V_{GS} - V_T) + \frac{C_{ox}}{q} NOIC(V_{GS} - V_T)^2 \right)
\]

Using above equation and taking into account the previous NOIB parameters we can deduce the NOIC parameters. Then the extracted mean value are respectively:

\[
NOIC = 1 \times 10^{20} V^{-1} m^{-2} \quad \text{For PMOS}
\]

\[
NOIC = 1.3 \times 10^{20} V^{-1} m^{-2} \quad \text{For NMOS}
\]

Fig 3. variation of the parameter NOIB vs the effective gate voltage
How to modeling for BSIM3V3 (1/f Noise)

- Model verification
  Noise measure: from subthreshold to strong inversion at $V_{ds}=4V$.
  Measured data are compared to simulated ones provided by the below equation:

$$S_{id} = \frac{qkT_{d}I_{eff}}{I_{off}C_{ox}f_{EF}} \left[ NOIA \ln \frac{N_{0} + N^{*}}{N_{L} + N^{*}} + NOIB(N_{0} - N_{L}) + \frac{NOIC}{2}(N_{0}^{2} - N_{L}^{2}) \right] + \Delta L_{clm} \frac{kT_{d}^{2}}{qW_{off}I_{off}f_{EF}} \frac{[NOIA + NOIBN_{L} + NOICN_{L}^{2}]}{N_{L}^{2}}$$

The transistor is biased in saturation regime, we take into account the influence of the reduction in the electrical channel length by fitting the “Litl” parameter.

$$Litl = 1.3 \times 10^{-7} m \quad \text{For PMOS}$$

$$Litl = 1.1 \times 10^{-7} m \quad \text{For NMOS}$$

Fig 4. experiment vs simulation (p type)

Fig 5. experiment vs simulation (n type)
Advanced Noise Model

- Quantitative analysis of the improved flicker noise model
  Hot electron stressing

Reference paper: Improved Flicker noise model for submicron mosfet devices

Theory
1) hot-carrier stressing degrades the operating lifetime of the devices
2) The high electric field (Emax) heats up and accelerates the electrons in the pinch-off region → generate the EHP
3) Generated electron are injected into the gate oxide.
   → increasing the number of filled oxide traps → higher 1/f spectral density

Fig 1. before stressing
Fig 2. after stressing

0.35um device
Vdd=3V
30 minutes stressing
Advanced Noise Model

\[ S_{id} = \frac{q^2 k T_i d \mu_{eff}}{a y L^2 C_{ox}} \left[ A \ln \frac{N_0 + N^*}{N_L + N^*} + B(N_0 - N_L) + \frac{C}{2} (N_0^2 - N_L^2) \right] + \frac{k T_i d \Delta L}{\gamma \ast f \ast L^2 \ast W} \left[ A + B N_L + C N_L^2 \right] \]

Two modifications
1) The increase in generated interface traps
2) The shift in threshold voltage.

\[ S_{id} = \frac{q^2 k T_i d \mu_{eff}}{a y L^2 C_{ox}} \left[ A \ln \frac{N_0 + N^*}{N_L + N^*} + B(N_0 - N_L) + \frac{C}{2} (N_0^2 - N_L^2) \right] + \frac{k T_i d \Delta L}{\gamma \ast f \ast L^2 \ast W} \left[ A_x + B_x N_{Dx} + C_x N_{Dx}^2 \right] \]

→ Final improved noise model

\( A_x, B_x, C_x \): generated oxide traps imply a higher oxide trap density \( N_t \) and this is reflected in new parameters

\( N_{Dx} \): Vth shift explain

\[ N_{Dx} = \frac{C_{ox}}{q} (V_{gs} - V_{th} - a V_{ds} - \Delta V_{th}) \]
Advanced Noise Model

Input referred noise

\[ S_{Vg} = \frac{S_{ld}}{g_m^2} \]

where

\[ g_m = WC_{ox}v_{sat}(V_{gs} - V_{th}) \left( \frac{V_{gs} - V_{th} + 2E_{sat}L}{V_{gs} - V_{th} + E_{sat}L} \right)^2 \]

\[ S_{Vg} = \frac{kT\Delta L}{\gamma \cdot f \cdot L^2 \cdot W} \left[ A_x + B_x N_{Dx} + C_x N_{Dx}^2 \right] \left( \frac{V_{gs} - V_{th} + E_{sat}L}{V_{gs} - V_{th} + 2E_{sat}L} \right)^2 \]

\[ \rightarrow \text{Last term in typically ranges between 0.27 and 0.45} \]

Fig 3. comparison of measured data with improved 1/f noise model before stressing

Technology 0.35um
CMOS process
Advanced Noise Model

**Fig 4.** Comparison of measured data with improved 1/f noise model after stressing. 1/f noise overshoot is due to hot-carrier stressing.

**Fig 5.** Comparison of input referred noise voltage. The gate bias dependence of the noise in submicron devices is accurately modeled by the improved model.
1/f noise with HiSIM model

- A new 1/f noise model of MOSFETs for circuit simulation down to 100nm Tech.
  Reference paper: Modeling of 1/f noise with HiSIM for 100nm CMOS technology

- Shortcoming of existing 1/f noise models
  1) Hardly reproduce the strong gate length dependence
  2) Hardly reproduce the bias dependence with a single model
  3) Large increase of noise by reducing the gate length
  4) Stronger channel length dependence than predicted by the conventional 1/LW linear relation

- HiSIM model developed !!
  1) Carrier density distribution along the channel
  2) 1/f noise valid for all gate lengths with a single parameter set
  3) Accuracy for any bias conditions and gate lengths with a single model parameter set
1/f noise with HiSIM model

Fig 1. drain current of nmos with different gate length under linear condition.

Fig 2. linear condition

Fig 3. saturation condition

- 1/f noise model Assumption
  Uniform trap density and energy distribution in the Oxide layer

Fig 1 and Fig 2 show that trap density and energy distribution is spatially non-uniform in the oxide layer!!
1/f noise with HiSIM model

The difference in the noise spectra between the Forward and backward measurement becomes Clear under the saturation

No difference in the measured drain current is Observed by exchange

→ Position dependent trap density and energy along the channel direction

Fig 4. saturation condition

Lorentzian Noise

\[ S_{ld} = \frac{A\tau}{1 + (2\pi/\tau)} \]

1) A is a magnitude of the Lorentzian noise determining Trap density
2) t is a time constant of the carriers in the G-R process
1/f noise with HiSIM model

Fig 5. Three dashed lines represent Ideal 1/f spectra and the dotted line in The results fitted with Lorentzian eqn

Fig 6. Length = 0.12um

Inhomogeneous trap site on the noise characteristics is enhanced due to the reduced gate length!!
1/f noise with HiSIM model

Lg=0.46um at f=100Hz

As a circuit-simulation model it is a subject to describe only this averaged 1/f noise characteristics with boundaries as the worst and the best case

Fig 7. By averaging the noise spectra over chips on a wafer
1/f noise with HiSIM model

- Model description

\[ S_{ld}(f) = \frac{kT I_d^2 N_{trap}}{qW L^2 f} \int_0^L \frac{1}{N(x) + N^*} (\pm \alpha \mu)^2 \, dx \]

where \( N^* = \frac{kT}{q^2} (C_{ox} + C_d + C_{it}) \)

\[ N_{trap} = N_t / \gamma \]

\( \gamma \) : Coefficient of the carrier fluctuation

\( N_{trap}, \alpha, C_{it} \) the ratio of the trap density to attenuation coefficient into the oxide.

To develop an precise 1/f noise model

1) Current Ids is important
2) Position dependent carrier concentration along the channel N(x)

→HiSIM provides the carrier concentrations at the source No and drain side NL determined by surface potentials consistently.

\[ S_{ld}(f) = \frac{kT I_d^2 N_{trap}}{qW L^2 f} \int_{\phi_0}^{\phi_L} \frac{1}{N(\phi) + N^*} (\pm \alpha \nu)^2 \, d\phi \]
1/f noise with HiSIM model

N(x) will be decreasing from No to NL

Fig 8. The inversion charge density at the source and drain side or pinch-off point in saturation mode

→ Length=1um

In the pinch-off region carriers loose the gate voltage control and number of carrier reduced
→ Diminished trapping/detrapping process

Fig 9. simulated number of channel electrons colliding with the oxide interface per unit time

→ Diminished noise power arises from the pinch-off region.
→ The L should be changed by

→ Length=0.12um
1/f noise with HiSIM model

Final analytical equation of the 1/f noise

\[
S_{ld}(f) = \frac{kT I_d^2 N_{trap}}{q W (L - \Delta L) f} \left( \frac{1}{(N_0 + N^*)(N_L + N^*)} + \frac{2\alpha\nu}{N_L - N_0} \log \left( \frac{N_L + N^*}{N_0 + N^*} \right) + (\alpha\nu)^2 \right)
\]

\(N_0, N_L\) are calculated by HiSIM

Fig 10. Comparison of the Vgs dependence of the measured and simulated drain current noise with various Length \((1\mu m, 0.46\mu m, 0.12\mu m)\) \(f=100Hz\)

\[
S_{ld}(f) = \frac{kT I_d^2 N_{trap}}{q W (L - \Delta L) f} \left( \frac{1}{N_{ave} + N^*} + (\alpha\nu)^2 \right)^2
\]

Average model \(N(x)\) model
1/f noise with HiSIM model

Average N(x) model cannot reproduce the bias dependences of the Sid for all channel lengths with a single model-parameter set.

Fig 11. Comparison of the Vds dependence of the measured and simulated drain current noise with various Length (1u, 0.46u, 0.12u) and fixed width=10um.

- The noise enhancement for larger Vds is not well reproduced...

Fig 12. Fixed Wg= 10um, f=100Mhz Length is varied.

- The well-confirmed 1/LW dependence
- But the deviation from the linear relationship is observed beyond Lg=0.14um
Noise measurement and modeling using UTMOST

- Silvaco Noise Box (S3245A Noise Amplifier)

**Specifications**

**Amplifier**

- Amplifier Type: Non-Inverting Voltage Amplifier
- Amplifier Gain: 121
- Amplifier Bandwidth: 300 kHz
- Amplifier Noise Floor: -128 dB (V)
- Frequency Range: 0.25 Hz - 1 MHz
- Load Resistance: 11 kΩ

**DUT**

- DUT Type: NMOS or PMOS transistors (wafer or packaged)
- Recommended Current range: +/- 10μA to +/- 10mA
- Voltage range: +/- 50V max
- Output impedance matching: 100Ω to 1MΩ
- Input impedance matching: 0 to 100MΩ

**System Requirements**

- DC Analyzer: HP4141, HP4142, HP4145, HP4155/56
- Dynamic Signal Analyzer: HP3561, HP3562, HP35660, HP35665, HP35670
- Computer Operating Systems: Solaris, Linux
- Software: UTMOST version 15.2.0 or later with Noise Module
- GPIB Interface: National Instruments GPIB-232CT-A Serial to GPIB converter Box
- Cables: 5 BNC cables, 4 Triax cables
Noise measurement and modeling using UTMOST

- Noise measurement and modeling using UTMOST

**SMU define**

- GPIB address

**S3245A Calibration**

- System serial port 1

**DSA instrument setup**

- GPIB Box setup
Noise measurement and modeling using UTMOST

- Hardware setup (UTMOST v.21.12.3.R) (DSA setup → 35670A)

- Vertical Units: In order to obtain $V^2/Hz$ for the spectrum density curves. This should be set to $VOLT^2$
- Fixed Scale Limit: Upper limit for the DSA’s vertical scale.
- MAG coordinate: Vertical scale setting for Linear or Log, Typical is Log
- Auto Scale: Auto scale for vertical scale after the measurement is finished
- Auto Cal: Allows DSA to calibrate itself when needed
- Single Cal: It runs a single calibration during the initialization process.
- # of Averages: the rms average is on. Typical setting is 10
- Start Freq (Hz): Measurement start frequency. Typical setting is 10
- Freq.span (Hz): the stop frequency = start freq + freq.span
- Freq.axis: Horizontal scale setting “ Linear or Log. Typical is Log
- Window: Typical setting is Uniform
- Coupling: DSA’s input coupling. AC or DC coupling is available.
- Run Setup: DSA Analyzer screen start the initialization process for the DSA. During the Run Setup operation, the DC Analyzer is not controlled
Noise measurement and modeling using UTMOST

- Hardware setup (UTMOST v.21.12.3.R) → Calibration of S3245A

Clear Cal → Setup Cal → "Calibration is successfully completed"
→ Check the Noise floor at DSA screen → Noise floor should be below -100db
→ If not satisfaction → Turn the light off → tried to re-calibration
Noise measurement and modeling using UTMOST

- Hardware setup (UTMOST v.21.12.3.R) \(\rightarrow\) setup screen

Select_model
KF extraction
NLEV=0
NLEV=1
NLEV=2

NOIA,NOIB,NOIC
For NOIMOD=2
Should be set to 3,4 for select_model
Noise measurement and modeling using UTMOST

### SILVACO Noise Models

<table>
<thead>
<tr>
<th>Noise Model</th>
<th>1/f noise</th>
<th>Thermal Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NLEV=0</strong></td>
<td>$S_{ld} = \frac{KF * I_d^{AF}}{f C_{ox} L_{eff}^2}$</td>
<td>$i_{dn}^2 = \left( \frac{8kT}{3} \right) g_m$</td>
</tr>
<tr>
<td><strong>NLEV=1</strong></td>
<td>$S_{ld} = \frac{KF * I_d^{AF}}{C_{ox} W L_{eff}} \frac{1}{f}$</td>
<td>$i_{dn}^2 = \left( \frac{8kT}{3} \right) g_m$</td>
</tr>
<tr>
<td><strong>NLEV=2</strong></td>
<td>$S_{ld} = \frac{KF * g_m^2}{C_{ox} W L_{eff}} \frac{1}{f^{AF}}$</td>
<td>$i_{dn}^2 = \left( \frac{8kT}{3} \right) g_m$</td>
</tr>
<tr>
<td><strong>NLEV=3</strong></td>
<td>$S_{ld} = \frac{KF * g_m^2}{C_{ox} W L_{eff}} \frac{1}{f^{AF}}$</td>
<td>$i_{dn}^2 = \left( \frac{8kT}{3} \right) \left[ BETA(V_{gs} - V_{th}) \frac{1 + a + a^2}{1 + a} GDSNOI \right]^{1/2}$</td>
</tr>
<tr>
<td><strong>NOIMOD=1</strong></td>
<td>$S_{ld} = \frac{KF * I_d^{AF}}{f \cdot EF C_{ox} L_{eff}^2}$</td>
<td>$i_{dn}^2 = \left( \frac{8kT}{3} \right) (g_m + g_{ds} + g_{mb})$</td>
</tr>
<tr>
<td><strong>NOIMOD=2</strong></td>
<td>$S_{ld} = \frac{qK T \mu_{eff}}{L_{ds} C_{ox}} \left[ NOIA \ln \left( \frac{N_0 + N_1}{N_1 + N_1'} + NOIB(N_0 - N_1) + \frac{NOIC}{2} (N_1^2 - N_1'^2) \right) \right] \ldots$</td>
<td>$i_{dn}^2 = \frac{4kT \mu_{eff}}{L_{eff}^2}</td>
</tr>
<tr>
<td><strong>NOIMOD=3</strong></td>
<td>$S_{ld} = \frac{qK T \mu_{eff}}{L_{ds} C_{ox}} \left[ NOIA \ln \left( \frac{N_0 + N_1}{N_1 + N_1'} + NOIB(N_0 - N_1) + \frac{NOIC}{2} (N_1^2 - N_1'^2) \right) \right] \ldots$</td>
<td>$i_{dn}^2 = \left( \frac{8kT}{3} \right) (g_m + g_{ds} + g_{mb})$</td>
</tr>
<tr>
<td><strong>NOIMOD=4</strong></td>
<td>$S_{ld} = \frac{KF * I_d^{AF}}{f \cdot EF C_{ox} L_{eff}^2}$</td>
<td>$i_{dn}^2 = \frac{4kT \mu_{eff}}{L_{eff}^2}</td>
</tr>
</tbody>
</table>
Noise measurement and modeling using UTMOST

- Hardware setup (UTMOST v.21.12.3.R) → setup screen

- VDS_start: Starting VDS
- VDS_step: VDS_step
- #_of_VDSstep: Number of step for VDS biasing
- VGS_start: Starting VGS
- #_of_VGSstep: Number of step for VGS biasing
- Amp_gain: S3245A amp gain (121)
- IDS_measured: Measured IDS current
- decade_sweep: The utmost will measure at each decade
- gm_measured: during the DC biasing of the MOS. The gm is measured
- gds_measured: during the DC biasing of the MOS. The gds is measured
- VDS_ext: S3245A had a load resistor in series to the MOS device’s drain. Due to the loading resistor the external VDS bias should be higher than the actual VDS applied to the device. UTMOST iterate the external VDS bias until the internal VDS is reached to the specified VDS
- debias_DC: if set to 0 the final DC bias conditions will be applied to the MOS device after the noise data is collected from the DSA. This is useful if the same measurement needs to be repeated manually
Noise measurement and modeling using UTMOST

- 1/f noise Modeling (UTMOST v.21.12.3.R) \(\rightarrow\) Measurement \((V^2/Hz)\)
Noise measurement and modeling using UTMOST

Noise measurement and modeling using UTMOST

- 1/f noise Modeling (UTMOST v.21.12.3.R) → Fitting (NOIMOD=2)

Run by: S.K. KIM
Process: CMOS
Temp: 27°C
Lot: 59S2150
Mask: 1
Die: 1

NOIA, NOIB, NOIC, EF, EM
Extracted

Optimization with External SmartSpice
Noise measurement and modeling using UTMOST

- Noise measurement and modeling using UTMOST

<table>
<thead>
<tr>
<th>Run by: S.K.KIN</th>
<th>Lot: 5588150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process: CMOS</td>
<td>RF: 1</td>
</tr>
<tr>
<td>Temp: 27</td>
<td>Ref: 1</td>
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</tbody>
</table>

![Graphs showing noise measurement and modeling using UTMOST](image)

<table>
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<tr>
<th>115</th>
<th>AF</th>
<th>1</th>
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</thead>
<tbody>
<tr>
<td>116</td>
<td>KF</td>
<td>8.234E-29</td>
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<tr>
<td>117</td>
<td>NOIMOD</td>
<td>2</td>
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<tr>
<td>116</td>
<td>NOIA</td>
<td>4.601279E17</td>
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<tr>
<td>119</td>
<td>NOIB</td>
<td>2.071198E4</td>
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<tr>
<td>120</td>
<td>NOIC</td>
<td>1E-13</td>
</tr>
<tr>
<td>121</td>
<td>EM</td>
<td>1.251976E4</td>
</tr>
<tr>
<td>122</td>
<td>EF</td>
<td>0.8087715</td>
</tr>
</tbody>
</table>
Noise measurement and modeling using UTMOST

- 1/f noise Modeling (UTMOST v.21.12.3.R) → Optimization (NLEV=3)

Target (Saturation mode)

<table>
<thead>
<tr>
<th>114</th>
<th>NLEV</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>115</td>
<td>$A_f$</td>
<td>0.8</td>
</tr>
<tr>
<td>116</td>
<td>$K_F$</td>
<td>8.23E-26</td>
</tr>
</tbody>
</table>

Target (Saturation mode)?

(Linear mode)
Thermal Noise Concept

- Thermal Noise Concept (Johnson Noise, Nyquist Noise)
  1) Thermal noise is the voltage fluctuations caused by the random Brownian motion of electrons in a resistive medium
  2) It is broadband white noise
  3) It increases with increasing resistance and temperature
  4) A fifty ohm resistor has about \( \frac{1}{2} nV / \sqrt{Hz} \) of thermal noise
  5) Thermal noise provides a current even in the absence of an external bias

(a) Ideal Resistor
- Non-physical resistor, carrier “randomly” collide with lattice atoms, giving rise to current variation over time

(b) Physical Resistor
- Can model random current component using a noise current source \( i(t) \)
Thermal Noise Concept

Current signal with period $T$, the average power is given by

$$ P_{av} = \frac{1}{T} \int_{-T/2}^{T/2} i^2(t) R * dt $$

Non-deterministic random process

$$ P_n = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} i_n^2(t) R * dt $$

$$ \overline{v_n^2} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} v_n^2(t) dt = \int_0^\infty |V_n(f)|^2 df $$

$$ \overline{i_n^2} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} i_n^2(t) dt = \int_0^\infty |I_n(f)|^2 df $$

Drop $R$ in the above expression because of Power equal to $i(t)*v(t)$
Thermal Noise Concept

PSD shows how much power a signal caries at a particular frequency

About 10% drop at 2Ghz

\[ P_n = \int_{f_1}^{f_2} PSD(f) df \]

Nyquist showed that the noise PSD of a resistor is

\[ PSD(f) = n_o = 4kT \]

\( k \) is the Boltzmann constant and \( T \) is the absolute temperature

\[ 4kT = 1.66 \times 10^{-20} \]
Thermal Noise Concept

The total average noise power of resistor in a certain frequency band is

\[ P_n = \int_{f_1}^{f_2} 4kT \, df = 4kT (f_2 - f_1) = 4kT \Delta f \]

Noise can be calculated using either an equivalent voltage or current generator

\[ \overline{v_n^2} = P_n \cdot R = 4kT \cdot R \cdot \Delta f \]

\[ \overline{i_n^2} = \frac{P_n}{R} = \frac{4kT}{R} \cdot \frac{1}{\Delta f} \]

For \( R = 1k\Omega \):

\[ \frac{\overline{v_n^2}}{\Delta f} = 16 \cdot 10^{-18} \frac{V^2}{Hz} \]

\[ \sqrt{\frac{\overline{v_n^2}}{\Delta f}} = 4nV / \sqrt{Hz} \]

\[ \sqrt{\frac{\overline{i_n^2}}{\Delta f}} = 4pA / \sqrt{Hz} \]
Thermal Noise Concept

- Two Resistor in series

\[
\bar{v}_n^2 = \bar{v}_{n1}^2 + \bar{v}_{n2}^2 = 4kT(R_1 + R_2)\Delta f
\]

\[
\bar{v}_n^2 = (\bar{v}_{n1} - \bar{v}_{n2})^2 = \bar{v}_{n1}^2 + \bar{v}_{n2}^2 - 2\bar{v}_{n1} \cdot \bar{v}_{n2}
\]

→ Uncorrelated signal

KT/C noise (Low pass filter)

MOS saturation mode
Thermal Noise Concept

- Low pass filter

\[ V_{out}(f) = V_{nt}(f) \ast H(f) \]

\[ \overline{V_{out}^2} = \int_0^\infty |V_{out}(f)|^2 df = \int_0^\infty |H(f)|^2 \ast |V_{nt}(f)|^2 df \]

\[ = 4kTR \int_0^\infty |H(f)|^2 df = 4kTR \int_0^\infty \frac{1}{1 + (f/f_c)^2} df \]

\[ \overline{V_{out}^2} = \frac{kT}{C} \quad \therefore \int_0^\infty \frac{1}{1 + x^2} dx = \frac{\pi}{2} \quad f_c = \frac{1}{2\pi RC} \]
MOS Thermal Noise

- MOSFET thermal noise model (SPICE2)

\[
|I_{dn}(f)|^2 = \frac{4kT}{R_{av}} \Delta f
\]

\[
R_{av} = \frac{L}{W\sigma_{av}}
\]

\[
\sigma_{av} = \frac{1}{L} \int_0^L \sigma(y)dy = \frac{1}{L} \int_0^L \mu_n q_N(y)dy
\]

\[
q_N(y) = C_{ox}(V_{GS} - V_{TH})\sqrt{1 - (y/L)}
\]

\[
\Rightarrow \sigma_{av} = \frac{2}{3} \mu_n C_{ox}(V_{GS} - V_{TH})
\]

\[
R_{av} = \frac{3L}{2W} \frac{1}{\mu_n C_{ox}(V_{GS} - V_{TH})}
\]

\[
|I_{dn}(f)|^2 = \frac{4kT}{R_{av}} \Delta f
\]

\[
|I_{dn}(f)|^2 = \left(\frac{8kT}{3}\right) g_m = 4kT \frac{2}{3} g_m \Delta f
\]

\[
\Rightarrow \text{Old model}
\]
MOS Thermal Noise

- New Model for the thermal noise
  \[ |I_{dnt}(f)|^2 = \frac{2}{3} 4kT(g_m + g_{mb})\Delta f \quad \rightarrow \text{PSD in saturation} \]

- Shortcoming
  1) This expression is incomplete for the saturation
  2) It can’t be used in the triode region. \( \rightarrow \) when for \( V_{ds} \rightarrow 0 \) it gives a value of thermal noise equal to zero

  \( \rightarrow \) The correct expression for the noise has to take into account the effect of the conductance due to channel modulation in saturation

  \[ |I_{dnt}(f)|^2 = \frac{2}{3} 4kT(g_m + g_{mb} + g_{ds})\Delta f \quad \rightarrow \text{SPICE2 model} \]

  \( \rightarrow \) for \( V_{ds} \rightarrow 0 \) the thermal noise depends on the channel conductance

  \[ |I_{dnt}(f)|^2 = 4kTg_{d0}\Delta f \quad \text{where} \quad g_{d0} = \left. \frac{\partial I_{ds}}{\partial V_{ds}} \right|_{V_{gs} = V_{by} = \text{const}, V_{ds} = 0} \]
MOS Thermal Noise

Limit condition for all operation regions is valid for \( 0 < V_{ds} < V_{dsat} \)

\[
| I_{dnt}(f) |^2 = \frac{2}{3} 4kT g_{eq}(V_{ds}) \Delta f
\]

\[ g_{eq}(V_{ds}) = (g_m + g_{mb} + g_{ds}) \left( \frac{3}{2} - \frac{V_{ds}}{V_{dsat}} \right) \]

\[ \text{◆ Using above equation} \]

\[
| I_{dnt}(f) |^2 = \frac{2}{3} 4kT (g_m + g_{mb} + g_{ds}) \left( \frac{3}{2} - \frac{V_{ds}}{V_{dsat}} \right) \Delta f \quad \text{if} \quad V_{ds} < V_{dsat}
\]

\[
| I_{dnt}(f) |^2 = \frac{2}{3} 4kT (g_m + g_{mb} + g_{ds}) \Delta f \quad \text{if} \quad V_{ds} > V_{dsat}
\]

What’s “2/3” means in thermal noise model?

\[
| I_{dnt}(f) |^2 = 4kT \gamma g_m \Delta f
\]

For long channel MOSFET \( \gamma = 2/3 \)

For short channel MOSFET \( \gamma \approx 1 \)
MOS Thermal Noise

- BSIM3V3.2.2 or before Thermal Noise model

\[
| I_{dnt}(f) |^2 = \frac{4kT \mu_{eff}}{L_{eff}^2} | Q_{inv} | \Delta f \\
Q_{inv} = -w_{eff} L_{eff} C'_{ox} V_{gseff} (1 - \frac{A_{bulk}}{2(V_{gseff} + 2vt)}) V_{dseff}
\]

BSIM3V3.3 Thermal Noise model

\[
| I_{dnt}(f) |^2 = \frac{4kT}{R_{DS} + \frac{L_{eff}^2}{(\mu_{eff} | Q_{inv})}} \Delta f
\]

Noise Model Flag in BSIM3 model

<table>
<thead>
<tr>
<th>NOIMOD flag</th>
<th>Flicker Noise model</th>
<th>Thermal noise model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SPICE2</td>
<td>SPICE2</td>
</tr>
<tr>
<td>2</td>
<td>BSIM3V3</td>
<td>BSIM3V3</td>
</tr>
<tr>
<td>3</td>
<td>BSIM3V3</td>
<td>SPICE2</td>
</tr>
<tr>
<td>4</td>
<td>SPICE2</td>
<td>BSIM3V3</td>
</tr>
</tbody>
</table>
MOS Noise

- **SPICE2 1/f noise**

\[
\frac{i_{nD}^2}{f} = \frac{K_F \cdot I_d^{AF}}{f^{EF} C_{ox} L_{eff}^2} \Delta f 
\]

\[
K_{F_{NMOS}} = 2.0 \times 10^{-29} \text{ AF} 
\]

\[
K_{F_{PMOS}} = 1.5 \times 10^{-29} \text{ AF} 
\]

For 0.35um CMOS

KF is strongly dependent on technology

- **BSIM3V3 1/f noise**

\[
S_{ld} = \frac{k T I_d \mu_{eff}}{L_{eff} C_{ox} f^{EF}} \left[ \frac{NOIA N_0 + N^*}{N_L + N^*} + NOIB(N_0 - N_L) + \frac{NOIC}{2} (N_0^2 - N_L^2) \right] 
\]

\[
+ \Delta L_{clm} \frac{k T I_d^2}{q W_{eff} L_{eff} f^{EF}} \frac{N_L}{N_L^2} 
\]

\[
NOIA_{NMOS} = 6.4 \times 10^{20} 
\]

\[
NOIA_{PMOS} = 3.5 \times 10^{18} 
\]

\[
NOIB_{NMOS} = 8 \times 10^4 
\]

\[
NOIB_{PMOS} = 5.3 \times 10^3 
\]

\[
EF_{NMOS} = 0.9 \sim 1.2 
\]

\[
EF_{PMOS} = 0.9 \sim 1.2 
\]

\[
EM_{NMOS} = 4.1 \times 10^7 
\]

\[
EM_{PMOS} = 4.1 \times 10^7 
\]

For 0.35um CMOS
MOS Noise

- 1/f Noise Corner

\[
\frac{K F \cdot I_d \Delta f}{f_{co} C_{ox} L_{eff}^2} = 4kT \gamma g_m \Delta f
\]

\[
f_{co} = \frac{K F}{4kT \gamma} \frac{1}{L^2} \frac{1}{g_m / I_D}
\]

For example \( g_m / I_D = 10V^{-1}, \gamma = 1 \)

<table>
<thead>
<tr>
<th>( L = 0.35\mu m )</th>
<th>( f_{co,NMOS} )</th>
<th>( f_{co,PMOS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>192kHz</td>
<td>34kHz</td>
<td></td>
</tr>
</tbody>
</table>

| \( L = 1\mu m \) | 24kHz | 4kHz |

In more recent technologies, 1/f corner frequencies can be on the order of 10MHz.
Another Noise

- Another Noise source

Shot Noise (caused by current flowing across a junction):
the shot noise relates to the dc current flow across a certain potential barrier.

\[ I_{ns}^2 = 2qI_D \]

Generation-recombination Noise:
trapping centers in the bulk of the device can cause GR Noise

\[ C(f) = KB \frac{I^{AB}}{1 + \left(\frac{f}{FB}\right)^2} \]

Impact ionization noise:
this noise is generated in the impact ionization process. The amount of noise proportional to \( I_{sub} \). When the impact ionization noise dominates, nmos have more noise than pmos.
Noise Modeling

Analog/Mixed-Signal Simulation

Bipolar Noise
Measurement System Configuration

- Measurement System for 1/f Noise of MOS and Bipolar

![Circuit Diagram]

Battery \(\rightarrow\) DUT \(\rightarrow\) LNA \(\rightarrow\) Shielding chamber \(\rightarrow\) Spectrum Analyzer (HP35670A)

**RD** should be matching to \(g_{ds}\) or \(g_m\).
Bipolar Equivalent Circuit

LNA

-3db frequency is almost 16Mhz

Noise spectral density is $1.02nV/\sqrt{Hz}$

Spectrum Analyzer (HP35670A)
Bipolar Equivalent Circuit

- Equivalent Circuit

\[ I_{R,i}^2 = 4kT \frac{1}{R_i} \Delta f, \quad i = \text{emitter, base, collector} \]

\[ i_{nb}^2, i_{nc}^2 = 2qI_{b,c} \Delta f + KF \frac{I_{AF}^b}{f} \Delta f \]
Noise parameter extraction

- AF, KF and BF/EF Noise Parameter Extraction

Reference document: Agilent Technologies GmbH, Munich

(Noise modeling for semiconductor)

For the BJT models, the origin of the 1/f noise is the Base region. However, the effective 1/f current noise spectra density \([A^2/Hz]\) is measured at the Collector of the transistor. Therefore, the 1/f noise at the base has to be calculated first.

\[
S_{iB} = \frac{1}{\beta^2} S_{iC} \left[ \frac{A^2}{Hz} \right]
\]

1/f effective noise current at the Base

\[
\overline{i^2_{nB}} = KF \frac{I_{B}^{AF}}{f} \Delta f
\]

VBIC95 model

\[
\overline{i^2_{nB}} = KF \frac{I_{B}^{AF}}{f} \Delta f
\]

BF to fit the -10dB/decade slope of 1/f noise

By multiplying \(f\)

\[
S_{iB} * f = KF * I_{B}^{AF} \iff S_{iB}@1Hz = KF * I_{B}^{AF}
\]

\[
\begin{align*}
S_{iB} &= \frac{\overline{i^2_{nB}}}{1Hz} = KF \frac{I_{B}^{AF}}{f} \left[ \frac{A^2}{Hz} \right] \\
S_{iB} * f &= KF * I_{B}^{AF} \\
\therefore S_{iB}@1Hz &= KF * I_{B}^{AF}
\end{align*}
\]
Noise parameter extraction

\[ S_{iB@1\text{HZ}} = KF \ast I_B^{AF} \]

Apply a logarithmic conversion to the above formula

\[ \log_{10}(S_{iB@1\text{HZ}}) = \log_{10}(KF + AF \log_{10} I_B) \]

Interpreted as a linear function like

\[ y = a + bx \]

where

\[ y = \log_{10}(S_{eB@1\text{HZ}}) \]
\[ a = \log_{10}(KF) \]
\[ b = AF \]
\[ x = \log_{10}(I_B) \]

A linear regression applied (y-intersect ‘a’ and slope ‘b’)

\[ AF = b \]
\[ KF = 10^a \]
Noise parameter extraction

Measured noise current at the Collector

- iB=1μA, Vce=2V

- iB=1μA~5μA (5 different base current)
  Vce=2V(fixed)
The 1/f noise source of a bipolar transistor is located and modeled in the Base region. Therefore we have to divide the above obtained collector current noise spectral density $S_{ic}$ by $\beta^2$.

$$S_{ib} = \frac{1}{\beta^2} S_{ic} \left[ \frac{A^2}{Hz} \right]$$

Therefore, $S_{ib@1Hz} = KF \cdot I_B^{AF}$.

Obtained $S_{ib}$ at the Base.

1Hz values of the 1/f current noise spectra density.
Noise parameter extraction

Finally, we ready to draw the 1HZ base noise data points against the DC bias

\[ \log_{10}(S_{ib@1HZ}) = \log_{10}(KF) + AF \log_{10}(I_B) \]

\[ y = a + bx \]

\[ y = \log_{10}(S_{eB@1HZ}) \]

\[ a = \log_{10}(KF) \]

\[ b = AF \]

\[ x = \log_{10}(I_B) \]

Simulation results of the collector current noise spectra density
Noise parameter extraction

Reference Paper: Accurate extraction method for 1/f noise parameters used in gummel-poon type bipolar junction transistor models
Noise parameter extraction

Final measured and simulated power spectra densities of low frequency noise

<table>
<thead>
<tr>
<th>Type</th>
<th>DUT A</th>
<th>DUT B</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF</td>
<td>1.304</td>
<td>1.478</td>
</tr>
<tr>
<td>KF</td>
<td>64.73e-15</td>
<td>107.4e-15</td>
</tr>
</tbody>
</table>

Low frequency noise parameters for several transistors